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Full Length Research Paper

The multi-attribute group decision making method based on the interval grey linguistic variables

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The extended TOPSIS method is proposed to solve multi-attribute group decision-making problems which the attribute values take the form of interval grey linguistic variables and attribute weight is unknown. To begin with, the relative concepts of interval grey linguistic variables are defined; the operation rules, the properties, and the distance between the two interval grey linguistic variables are established. Then the evaluation information of each expert is aggregated into the group information by the arithmetic weighted average method, and the mathematical model is constructed to solve the attribute weight based on the rules of the maximum deviation. Furthermore, the ranking order of alternatives is determined by TOPSIS method. Finally, the practical example is given to show the decision-making steps and the effectiveness of this method.

Key words: Grey fuzzy number, interval grey linguistic variables, TOPSIS, multi-attribute group decision making.

INTRODUCTION

Multiple attribute decision making (MADM) has been extensively applied to various areas such as society, economics, management, military and engineering technology. For example, investment decision-making, project evaluation, economic evaluation, personnel evaluation etc. Since the object things are complex, uncertainty and Human thinking is ambiguous, the majority of multi-attribute decision- making is uncertain and fuzzy, so fuzziness is the major factor in the process of decision making. In dealing with the problem of incomplete information caused by poor information, decision-making demonstrated its greyness. Therefore, the decision making problems demonstrated not only its fuzziness, but also its greyness, which is called the grey fuzzy multi-attribute decision making problems. Grey fuzzy multi-attribute decision making is defined as the method by which deciding the things or phenomena with fuzzy factors under the premise of insufficient information which have already known. The "grey" means that objective uncertainty caused by the insufficient and incomplete information, while the "fuzzy" means that the uncertainty factors in the evaluation information, which is the fuzziness of human thinking. The two is not the description of the same concept (Bu and Zhang 2002).

The research on grey fuzzy decision making problems has got rich achievements. Grey Analysis method was firstly presented by Professor Deng Julong (Deng, 1989; 2002; 2003), and it was well applied in multiple attribute decision making. Bu and Zhang (2002); Jin and Lou (2003, 2004); Choobineh and Li (1993a; 1993b); Luo and Liu (2004) studied the ranking method of grey fuzzy number. Bu and Zhang (2002) transformed the grey fuzzy number into the interval number, and then utilized the ranking method of interval number to rank the order of alternatives. According to the grey fuzzy multiple attribute decision making problems which both the fuzzy part and the grey part took the form of real number, Jin and Lou (2003) proposed the decision making model which utilized the hamming distance to measure the alternatives and utilized the difference between the fuzzy positive ideal solution and the negative ideal solution to rank the orders. Jin and Lou (2004) utilized the distance between each alternative and the grey fuzzy ideal solution to rank the orders of alternatives. In order to solving the grey fuzzy decision making problems, Luo and Liu (2004) utilized the maximum entropy formulism to determine attribute weight, then ranked the orders of alternative based on the linear combination of fuzzy information and grey information. Zhu et al. (2006) constructed the evaluation model in

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which the fuzzy part and the grey part took the form of interval number and the real number respectively.

Meng et al. (2007) proposed to present greyness and fuzziness of grey fuzzy decision making problems with the interval numbers, and the mathematical model of interval valued grey fuzzy comprehensive evaluation is established. At last its application to the selection of the preferred project is given. Wang and Wang (2008) extended the grey fuzzy number which both the fuzzy part and the grey part took the form of interval number, and ranked the order of alternatives based on the OWA operator. Because the linguistic variable is easier to express fuzzy information, this paper proposed the concept of interval grey linguistic variables which the fuzzy part and the grey part took the form of linguistic variables and interval numbers respectively, then studied the operation rules and the multiple attribute decision making method based on interval grey linguistic variables.

PRELIMINARIES

The foundation of the grey fuzzy math (Chen, 1994; Wang et al., 1996; Li and Wang, 1994; Wang and Song, 1988)

Definition 1: Let A be the fuzzy subset in the space $X = \{x\}$, if the membership degree $\mu_A(x)$ of x to A is the grey in the interval [0, 1], and its grey is $\nu_A(x)$, then A is called the grey fuzzy set in space X:

$$\underset{\otimes}{A} = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\} \quad (1)$$

The set pair mode is $A_\otimes = \left(A, A_\otimes\right)$, where $A = \{(x, \mu_A(x)) \mid x \in X\}$ is called the fuzzy part of $\underset{\otimes}{A}$, and $\underset{\otimes}{A} = \{(x, \nu_A(x)) \mid x \in X\}$ is called the grey part of $\underset{\otimes}{A}$.

So the grey fuzzy set is regarded as the generalization of the fuzzy set and the grey set.

Definition 2: Let $X = \{x\}$ and $Y = \{y\}$ be the given space, if $\nu_R(x, y)$ is the grey of the membership function $\mu_R(x, y)$ of R which is the fuzzy relationship between x and y, then grey fuzzy set $\underset{\otimes}{R} = \{((x, y), \mu_A(x, y), \nu_A(x, y)) \mid x \in X, y \in Y\}$ is called the grey fuzzy relationship in direct product space $X \times Y$, which is represented as the grey fuzzy matrix mode:

$$\underset{\otimes}{R} = \begin{pmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & (\mu_{2n}, \nu_{2n}) \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & (\mu_{mn}, \nu_{mn}) \end{pmatrix} \quad (2)$$

And $R_\otimes = \left(R, R_\otimes\right)$ represents the grey fuzzy relationship in directproduct space $X \times Y$, where $R = \{((x, y), \mu_A(x, y)) \mid x \in X, y \in Y\}$ represents the fuzzy relationship in direct product space $X \times Y$, and $\underset{\otimes}{R} = \{((x, y), \nu_A(x, y)) \mid x \in X, y \in Y\}$ represents the grey relationship in direct product space $X \times Y$.

The linguistic evaluation set and its extension

Suppose that $S = (s_0, s_1, , s_{l-1})$ is a finite and totally ordered discrete term set, where l the odd number is. In practical situation, l is equal to 3,5,7,9 etc. In this paper, l =7. For example, a set S could be given as follows (Herrera and Herrera-Viedma, 2000):

$S = (s_0, s_1, s_2, s_3, s_4, s_5, s_6)$ = {very poor, poor, slightly poor, fair, slightly good, good, and very good}.

Usually, in these cases, it usually requires that s_i and s_j must satisfy the following additional characteristics:

(1) The set is ordered: $s_i \quad s_j$, if and only if $i \quad j$;

(2) There is the negation operator: $neg(s_i) = s_{-i}$, such that $j = l - i$;

(3) Maximum operator: $\max(s_i, s_j) = s_i$, if $s_i \ge s_j$;

(4) Minimum operator: $\min(s_i, s_j) = s_i$, if $s_i \le s_j$;

For any linguistic set $S = (s_0, s_1, , s_{l-1})$, the relationship between the element s_i and its subscript i is strictly monotone increasing (Herrera and Herrera-Viedma 1996; Xu, 2006a), so the function can be defined as follows:

$$f : s_i = f(i)$$

Obviously, the function $f(i)$ is the strictly monotone increasing function about subscript i. To preserve all the given information, the discrete linguistic label $S = (s_0, s_1, , s_{l-1})$ is extended to a continuous label

$s = \{s_\alpha \mid \alpha \in R\}$ which satisfied the above characteristics. The operations are defined as follow (Xu, 2006b):

$$\beta s_i = s_{\beta \times i} \quad (3)$$

$$s_i \oplus s_j = s_{i+j} \quad (4)$$

$$s_i \oplus s_j = s_j \oplus s_i \quad (5)$$

$$\lambda(s_i \oplus s_j) = \lambda s_i \oplus \lambda s_j \quad (6)$$

$$(\lambda_1 + \lambda_2) s_i = \lambda_1 s_i \oplus \lambda_2 s_i \quad (7)$$

Definition 3: Let s_α , s_β be the two linguistic variables, then we defined the distance between s_α and s_β as:

$$d(s_\alpha, s_\beta) = |\alpha - \beta| / (l-1) \quad (8)$$

THE INTERVAL GREY LINGUISTIC VARIABLES

The definition of the interval grey linguistic variables

Definition 4: Let $A_\otimes = (A, A_\otimes)$ be the grey fuzzy number, if its fuzzy part is a linguistic variable $s_\alpha \in S$ and its grey part $\otimes$ L U $\otimes$ the interval grey linguistic variables. Because the linguistic variable is easier to express fuzzy information, it is reasonable to utilize the linguistic variables to represent the fuzzy part. While the grey part which indicated the amount of information obtained is described by the interval number, which can truly reflect the information the decision maker obtained. The larger the greyness of the grey part is, the less the information obtained is, and the lower the credibility of the obtained information, that is, the lower the credibility of the obtained value is, the lower the usage value of the information is. When the greyness goes up to some extent, it means that the obtained information is useless. On the contrary, the smaller the greyness is, the more the obtained information is, the higher the credibility of the obtained information, the more credible the obtained value is, the higher the use value of the obtained information.

The operation of the interval grey linguistic variables

variables Supposed that $A_\otimes = \left(s_\alpha, g_A^L, g_A^U\right)$, $B_\otimes = \left(s_\beta, g_B^L, g_B^U\right)$ and $C_\otimes = \left(s_\lambda, g_C^L, g_C^U\right)$ be the three interval grey linguistic variables. Based on the concept of the interval grey linguistic variables, the linguistic operational rules and extension principle, the operation rules of interval grey linguistic variables is defined as follows:

$$A_\otimes + B_\otimes = \left(s_{\alpha+\beta}, \left(1-(1-g_A^L)\times(1-g_B^L)\right), \left(1-(1-g_A^U)\times(1-g_B^U)\right)\right) \quad (9)$$

$$A_\otimes - B_\otimes = \left(s_{\alpha-\beta}, \left(1-(1-g_A^L)\times(1-g_B^L)\right), \left(1-(1-g_A^U)\times(1-g_B^U)\right)\right) \quad (10)$$

$$A_\otimes \times B_\otimes = \left(s_{\alpha\times\beta}, \left(1-(1-g_A^L)\times(1-g_B^L)\right), \left(1-(1-g_A^U)\times(1-g_B^U)\right)\right) \quad (11)$$

$$A_\otimes / B_\otimes = \left(s_{\alpha/\beta}, \left(1-(1-g_A^L)\times(1-g_B^L)\right), \left(1-(1-g_A^U)\times(1-g_B^U)\right)\right) \quad (12)$$

$$k A_\otimes = \left(s_{k\times\alpha}, g_A^L, g_A^U\right) \quad (13)$$

$$\left(A_\otimes\right)^k = \left(s_{\alpha^k}, g_A^L, g_A^U\right) \quad (14)$$

It can be seen that the interval gray linguistic variables satisfied the following properties

$$A_\otimes + B_\otimes = B_\otimes + A_\otimes \quad (15)$$

$$A_\otimes \times B_\otimes = B_\otimes \times A_\otimes \quad (16)$$

$$(A_\otimes + B_\otimes) + C_\otimes = A_\otimes + (B_\otimes + C_\otimes) \quad (17)$$

$$(A_\otimes \times B_\otimes) \times C_\otimes = A_\otimes \times (B_\otimes \times C_\otimes) \quad (18)$$

$$A_\otimes \times (B_\otimes + C_\otimes) = A_\otimes \times B_\otimes + A_\otimes \times C_\otimes \quad (19)$$

$$(\lambda_1 + \lambda_2) A_\otimes = \lambda_1 A_\otimes + \lambda_2 A_\otimes \quad (20)$$

The distance between the two interval grey linguistic variables

Definition 5: Let $A_\otimes$ $B_\otimes$ $\otimes$ C be the interval gray linguistic variables, $Z_\otimes$ be the set of the interval gray linguistic variables, f be the mapping, $f: Z_{\otimes} \times Z_{\otimes} \to R$ if $d\left(A_{\otimes}, B_{\otimes}\right)$, and satisfied the following formula:

(1) $0 \le d\left(A_{\otimes}, B_{\otimes}\right) \le 1$ $d\left(A_{\otimes}, A_{\otimes}\right) = 0$

(2) $d\left(A_{\otimes}, B_{\otimes}\right) = d\left(B_{\otimes}, A_{\otimes}\right)$

(3) $d\left(A_{\otimes}, B_{\otimes}\right) + d\left(B_{\otimes}, C_{\otimes}\right) \ge d\left(A_{\otimes}, C_{\otimes}\right)$

Then $d\left(A_{\otimes}, B_{\otimes}\right)$ is called the distance between the interval gray linguistic variable $A_{\otimes}$ and $B_{\otimes}$.

Definition 6: Let $A_{\otimes} = \left(s_{\alpha}, g_A^L, g_A^U\right)$ and $B_{\otimes} = \left(s_{\beta}, g_B^L, g_B^U\right)$ be the interval gray linguistic variables, then the Hamming distance $d\left(A_{\otimes}, B_{\otimes}\right)$ between the interval gray linguistic variable $A_{\otimes}$ and $B_{\otimes}$ is defined as follows:

$$d\left(A_{\otimes}, B_{\otimes}\right) = \frac{1}{2(l-1)}\left(\left|\alpha\left(1-g_A^L\right)-\beta\left(1-g_B^L\right)\right| + \left|\alpha\left(1-g_A^U\right)-\beta\left(1-g_B^U\right)\right|\right) \quad (21)$$

Proof: Formula (21) satisfied Conditions (1) and (2) of Definition 5 obviously. Now verified, the Formula (21) also satisfied Condition (3) of Definition 5.

For any interval gray linguistic variable $C_{\otimes} = \left(s_{\lambda}, g_C^L, g_C^U\right)$, we can get:

$$d\left(A_{\otimes}, C_{\otimes}\right) = \frac{1}{2(l-1)}\left(\left|\alpha\left(1-g_A^L\right)-\lambda\left(1-g_C^L\right)\right| + \left|\alpha\left(1-g_A^U\right)-\lambda\left(1-g_C^U\right)\right|\right)$$

$$= \frac{1}{2(l-1)}\left(\left|\alpha\left(1-g_A^L\right)-\beta\left(1-g_B^L\right)+\beta\left(1-g_B^L\right)-\lambda\left(1-g_C^L\right)\right| + \left|\alpha\left(1-g_A^U\right)-\beta\left(1-g_B^U\right)+\beta\left(1-g_B^U\right)-\lambda\left(1-g_C^U\right)\right|\right)$$

$$\le \frac{1}{2(l-1)}\left(\left|\alpha\left(1-g_A^L\right)-\beta\left(1-g_B^L\right)\right|+\left|\beta\left(1-g_B^L\right)-\lambda\left(1-g_C^L\right)\right| + \left|\alpha\left(1-g_A^U\right)-\beta\left(1-g_B^U\right)\right|+\left|\beta\left(1-g_B^U\right)-\lambda\left(1-g_C^U\right)\right|\right)$$

While;

$$\frac{1}{2(l-1)}\left(\left|\alpha\left(1-g_A^L\right)-\beta\left(1-g_B^L\right)\right|+\left|\beta\left(1-g_B^L\right)-\lambda\left(1-g_C^L\right)\right| + \left|\alpha\left(1-g_A^U\right)-\beta\left(1-g_B^U\right)\right|+\left|\beta\left(1-g_B^U\right)-\lambda\left(1-g_C^U\right)\right|\right)$$

$$= \frac{1}{2(l-1)}\left(\left|\alpha\left(1-g_A^L\right)-\beta\left(1-g_B^L\right)\right| + \left|\alpha\left(1-g_A^U\right)-\beta\left(1-g_B^U\right)\right| + \left|\beta\left(1-g_B^L\right)-\lambda\left(1-g_C^L\right)\right| + \left|\beta\left(1-g_B^U\right)-\lambda\left(1-g_C^U\right)\right|\right)$$

$$= d\left(A_{\otimes}, B_{\otimes}\right) + d\left(B_{\otimes}, C_{\otimes}\right)$$

So, $d\left(A_{\otimes}, B_{\otimes}\right) + d\left(B_{\otimes}, C_{\otimes}\right) \ge d\left(A_{\otimes}, C_{\otimes}\right)$.

Specially, if $g_A^L = g_A^U = g_B^L = g_B^U = 0$, then the interval gray linguistic variable is reduced to linguistic variable, and the Formula (21) is transformed into Formula (8). That is, the Formula (8) is the special case of Formula (21).

THE MULTIPLE ATTRIBUTE GROUP DECISION MAKING METHOD BASED ON THE INTERVAL GREY LINGUISTIC VARIABLES

The description of the multiple attribute group decision making problem based on the interval grey linguistic variables

Let $E = \left\{e_1, e_2, \ldots, e_p\right\}$ be the experts set in the group decision making, $A = \left\{A_1, A_2, \ldots, A_m\right\}$ be the set of alternatives, and $C = \left\{C_1, C_2, \ldots, C_n\right\}$ be the attribute set with respect to the alternatives. Supposed that $A_{\otimes ij}^k = \left(t_{ij}^k, g_{ijk}^L, g_{ijk}^U\right)$ be the attribute value in the attribute set C_j with respect to the alternative A_i, given by expert e_k, and $A_{\otimes}^k = \left(A_{\otimes ij}^k\right)_{m\times n}$ be the decision making matrix given by the expert e_k, where t_{ij}^k is the fuzzy part of the interval grey linguistic variables $A_{\otimes ij}^k$, $t_{ij}^k \in S$, S is the linguistic label, is the grey part of interval grey linguistic variables $A_{\otimes ij}^k$. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ be the experts weight, where $\sum_{k=1}^{p}\lambda_k = 1$. The attribute weight is unknown. Ranking the order of the alternatives, based on the experts weight and interval grey linguistic variables in each attribute of each alternative, given by each expert.

The decision making steps

Aggregate the evaluation information of each expert

Based on attribute value $A_{\otimes ij}^k$ in each attribute of each alternative, given by each expert, the decision making matrix $A_{\otimes}^k$ given by each experts are aggregated into group matrix $X_\otimes$, where $X_\otimes = (X_{\otimes ij})_{m\times n}$, and $X_{\otimes ij} = (t_{ij}, [g_{ij}^L, g_{ij}^U])$. According to the arithmetic weighted average method, we can get $X_{\otimes ij} = \sum_{k=1}^{p} \lambda_k A_{\otimes ij}^k$, where;

$$t_{ij} = \sum_{k=1}^{p} \left(\lambda_k t_{ij}^k\right) \tag{22}$$

$$[g_{ij}^L, g_{ij}^U] = \left[1-\prod_{k=1}^{p}(1-g_{ijk}^L),\ 1-\prod_{k=1}^{p}(1-g_{ijk}^U)\right] \tag{23}$$

Determine the attribute weight based on the rules of the maximum deviation

While the attribute weights are unknown, the uncertainty of attribute weight causes the uncertainty of ranking the alternative orders. In general, if the attribute values $X_{\otimes ij}$ among all the alternatives have smaller deviation with respect to attribute C_j, it shows that the attribute plays a less important role in the decision-making procedure. Contrariwise, if the attribute C_j makes the attribute values $X_{\otimes ij}$ among all the alternatives have larger deviation, such an attribute plays an important role in choosing the best alternative. So to the view of sorting the alternatives, if the attribute has similar attribute values across alternatives, it should be assigned a smaller weight; otherwise, the attribute which makes larger deviations should be evaluated a bigger weight. For the attribute C_j, the deviation value of alternative A_i to all the other alternatives can be defined as $D_{ij}(w) = \sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj}) w_j$, $D_j(w_j) = \sum_{i=1}^{m} D_{ij}(w) = \sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj}) w_j$ represents the total deviation value of all alternatives to the other alternatives for the attribute. $D(w) = \sum_{j=1}^{n} D_j(w_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj}) w_j$ represents the deviation of all attributes to all alternatives.

The maximum deviation model can be constructed as follows:

$$\max D(w) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj}) w_j$$
$$s.t \ \sum_{j=1}^{n} w_j^2 = 1,\ w_j \ge 0,\ j = 1, 2\ldots\ldots n \tag{24}$$

According to calculating, we can get:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj})}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj})\right)^2}} \tag{25}$$

After normalizing, we can get:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(X_{\otimes ij}, X_{\otimes lj})} \tag{26}$$

Rank the orders of alternatives by TOPSIS method

Calculate the ideal solutions of each alternative: The matrix $X_\otimes = (X_{\otimes ij})_{m\times n}$ is interval grey linguistic variables decision making matrix where $X_{\otimes ij} = (t_{ij}, [g_{ij}^L, g_{ij}^U])$, and the attribute vector of the positive ideal solution V^+ which belongs to its alternatives is:

$$V_\otimes^+ = (V_{\otimes 1}^+, V_{\otimes 2}^+, \ldots, V_{\otimes n}^+) \tag{27}$$

Where $V_{\otimes j}^+ = (z_j^+, [h_j^{+L}, h_j^{+U}])$, then:

$$z_j^+ = \max_i(t_{ij}),\ h_j^{+L} = \min_i(g_{ij}^L),\ h_j^{+U} = \min_i(g_{ij}^U)$$

Similarly, the attribute vector of the negative ideal solution V^- is:

$$V_{\otimes}^{-} = \left(V_{\otimes 1}^{-}, V_{\otimes 2}^{-}, \ldots, V_{\otimes n}^{-}\right) \quad (28)$$

Where $V_{\otimes j}^{-} = \left(z_j^{-}, h_j^{-L}, h_j^{-U}\right)$, then:

$$z_j^{-} = \min_i \left(t_{ij}\right), h_j^{-L} = \max_i \left(g_{ij}^{L}\right), h_j^{-U} = \max_i \left(g_{ij}^{U}\right)$$

Calculate the weighted distance between each alternative A_i and the ideal solutions:

$$D^{+}\left(A_i, V^{+}\right) = \sum_{j=1}^{n} w_j a \left(X_{\otimes ij}, V_{\otimes j}^{+}\right) \quad (29)$$

$$D^{-}\left(A_i, V^{-}\right) = \sum_{j=1}^{n} w_j a \left(X_{\otimes ij}, V_{\otimes j}^{-}\right) \quad (30)$$

Where $d\left(X_{\otimes ij}, V_{\otimes j}^{+}\right)$ represents the distance between the interval grey linguistic variables $X_{\otimes ij}$ and the positive ideal solution $V_{\otimes j}^{+}$, and $d\left(X_{\otimes ij}, V_{\otimes j}^{-}\right)$ represents the distance between the intervals grey linguistic variables $X_{\otimes ij}$ and the negative ideal solution $V_{\otimes j}^{-}$.

Calculate the relative closeness of each alternative:

$$Q\left(A_i\right) = \frac{D^{+}\left(A_i, V^{+}\right)}{D^{+}\left(A_i, V^{+}\right) + D^{-}\left(A_i, V^{-}\right)} \quad (31)$$

Rank the alternatives: A set of alternatives can now be ranked according to the descending order of $Q\left(A_i\right)$ and the one with the minimum value of $Q\left(A_i\right)$ is the best.

PRACTICAL EXAMPLES

A practical use of the proposed approach involves the evaluating the technological innovation ability of the four enterprises $\{A_1, A_2, A_3, A_4\}$, the attributes is shown as follows: the ability of innovative resources input (C_1), the ability of innovation management (C_2), the ability of innovation tendency (C_3) and the ability of research and development (C_4). Based on the four attributes, the three experts $\{e_1, e_2, e_3\}$ evaluated the technological innovation ability of the four enterprises. Supposed that $\lambda = (0.4, 0.32, 0.28)$ be the weight vector given by the three experts, and the attribute values given by the experts take the form of interval grey linguistic variables, shown in Tables 1, 2 and 3. Let $S = (s_0, s_1, s_2, s_3, s_4, s_5, s_6)$ be the linguistic label and the attribute weight be unknown. The problem is ranking the four enterprises based on their technological innovation ability. The evaluation steps used in this paper are proposed as follows:

(1) Based on Formulae (22) and (23), aggregate the evaluation information (Tables 1, 2 and 3) given by the experts $\{e_1, e_2, e_3\}$, then we can get the group decision making matrix $X_{\otimes}$:

$$X_{\otimes} = \begin{pmatrix} (s_{4.68},[0.42,0.71]) & (s_{2.60},[0.66,0.71]) & (s_{4.08},[0.76,0.80]) & (s_{4.24},[0.62,0.79]) \\ (s_{4.32},[0.75,0.79]) & (s_{4.36},[0.71,0.82]) & (s_{3.04},[0.35,0.62]) & (s_{3.40},[0.64,0.72]) \\ (s_{3.60},[0.49,0.71]) & (s_{4.28},[0.55,0.71]) & (s_{2.52},[0.62,0.66]) & (s_{4.08},[0.55,0.66]) \\ (s_{4.84},[0.72,0.83]) & (s_{2.92},[0.57,0.72]) & (s_{2.96},[0.61,0.78]) & (s_{3.88},[0.66,0.82]) \end{pmatrix}$$

(2) Calculate the attribute weight based on the rules of the maximum deviation

$$W = (0.34, 0.22, 0.23, 0.21)$$

(3) Calculate the ideal solutions

$$V_{\otimes}^{+} = \left((s_{4.84},[0.42,0.71]), (s_{4.36},[0.55,0.71]), (s_{4.08},[0.35,0.62]), (s_{4.24},[0.55,0.66])\right)$$

$$V_{\otimes}^{-} = \left((s_{3.60},[0.75,0.83]), (s_{2.60},[0.71,0.82]), (s_{2.52},[0.76,0.80]), (s_{3.40},[0.66,0.82])\right)$$

(4) Calculate the distance between each alternative and the ideal solutions

$$D^{+} = (0.42, 0.47, 0.33, 0.58)$$

Table 1. The attribute values of each attribute with respect to four enterprises given by epert e_1 .

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
A1	$(s_5,[0.2,0.3])$	$(s_2,[0.4,0.4])$	$(s_5,[0.5,0.5])$	$(s_3,[0.2,0.4])$
A2	$(s_4,[0.4,0.4])$	$(s_5,[0.4,0.5])$	$(s_3,[0.1,0.2])$	$(s_4,[0.5,0.5])$
A3	$(s_3,[0.2,0.3])$	$(s_4,[0.2,0.3])$	$(s_4,[0.3,0.3])$	$(s_5,[0.2,0.3])$
A4	$(s_6,[0.5,0.6])$	$(s_2,[0.2,0.2])$	$(s_3,[0.2,0.4])$	$(s_3,[0.3,0.4])$

Table 2. The attribute values of each attribute with respect to four enterprises given by expert e_2 .

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
A1	$(s_4,[0.1,0.3])$	$(s_3,[0.2,0.3])$	$(s_3,[0.2,0.2])$	$(s_6,[0.4,0.5])$
A2	$(s_5,[0.4,0.5])$	$(s_3,[0.3,0.4])$	$(s_4,[0.2,0.4])$	$(s_3,[0.2,0.3])$
A3	$(s_4,[0.2,0.4])$	$(s_4,[0.2,0.3])$	$(s_2,[0.4,0.4])$	$(s_3,[0.3,0.3])$
A4	$(s_5,[0.3,0.4])$	$(s_4,[0.4,0.5])$	$(s_2,[0.3,0.4])$	$(s_4,[0.2,0.4])$

Table 3. The attribute values of each attribute with respect to four enterprises given by expert e_3 .

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
A1	$(s_5,[0.2,0.4])$	$(s_3,[0.3,0.3])$	$(s_4,[0.4,0.5])$	$(s_4,[0.2,0.3])$
A2	$(s_4,[0.3,0.3])$	$(s_5,[0.3,0.4])$	$(s_2,[0.1,0.2])$	$(s_3,[0.1,0.2])$
A3	$(s_4,[0.2,0.3])$	$(s_5,[0.3,0.4])$	$(s_1,[0.1,0.2])$	$(s_4,[0.2,0.3])$
A4	$(s_3,[0.2,0.3])$	$(s_3,[0.1,0.3])$	$(s_4,[0.3,0.4])$	$(s_5,[0.4,0.5])$

$$D^- = (0.37,0.31,0.46,0.20)$$

(5) Calculate the relative closeness of each alternative

$$Q = (0.53,0.60,0.42,0.74)$$

(6) Rank the alternatives.

So we can get the orders of technological innovation ability of the four enterprises $\{A_1, A_2, A_3, A_4\}$:

$A_3 \quad A_1 \quad A_2 \quad A_4$.

Conclusions

Real decision making problems have not only the fuzziness but also the greyness, so the study on grey fuzzy multiple attribute decision making is very significant. Because the linguistic variable is easier to express fuzzy information, this paper proposed the concept of interval grey linguistic variables which the fuzzy part and the grey part took the form of linguistic variables and interval numbers respectively, then studied the multiple attribute group decision making method based on interval grey linguistic variables, and proposed the decision making steps. This method which proposed in this paper is easy to use and understand enriched and developed the theory and method of grey fuzzy multiple attribute decision making, and provided the new idea to solve the grey fuzzy multiple attribute decision making.

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