

Full Length Research Paper

# Modelling the volatility of exchange rates in the Kenyan market

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**This paper considers the application of the generalized autoregressive conditional heteroscedasticity process in the estimation of volatility in the Kenyan exchange rates. A quasi-maximum likelihood estimation procedure was used and asymptotic properties of the estimators were given. Exploratory data analysis performed indicated that the returns are heavy tailed. It was found that the estimated model fits the exchange rates return data well.**

**Key words:** Volatility, exchange, returns, autoregressive, heteroscedasticity, likelihood, quasi, maximum, estimator.

## INTRODUCTION

Volatility in exchange rates cannot be ignored in the exchange markets as both importers and exporters of goods and services are affected by exchange rate risk. Smith et al. (1990) found that volatility in prices has implications on the profits and survival of business enterprises. Exporters will remain in business as long as the export earnings can sustain their trade, while importers will maintain their businesses only if they can afford to buy foreign currency required to purchase goods.

Exchange rate depreciation could also have inflationary effects on an economy. Holmes (2003) found that exchange rate depreciation was inflationary although the impact could not prevail over the gains from increased external competitiveness. Depreciation reduces the real value of assets denominated in the local currency and increases the real value of foreign currency denominated assets. Assuming a constant money supply, domestic inflation increases if the first effect dominates the second effect. This brings to the fore the importance of reliable models for policy analysis and forecasting of the volatility of the exchange rates to guide central banks

on when to intervene in the foreign exchange rate market. Exchange rates in Kenya have witnessed significant volatility since liberalisation in October, 1993 (Njuguna et al., 2001), where factors that affect movements in the rates are analysed. The paper shows that an increase in the difference between domestic and foreign interest rates results in an appreciation of the exchange rate by attracting private capital flows. Similarly, improvements in the current account balance and net external inflows leads to an appreciation of the exchange rate (Calderon et al., 2001). However, although the exchange rate is expected to depreciate with a widening price differential, the study also found that key announcements, particularly by donors, affect the exchange rate.

Prolonged high volatility in exchange rate is an indication of ineffectiveness of a central bank to perform its core mandate of ensuring price stability, and management of the country's foreign exchange reserves. Therefore, the knowledge of volatility and its estimation can ensure mitigation of the long term risk of any investment, (Choy, 2002). This in turn assists in promoting economic growth, since investment is the main channel of increasing real output and employment. This paper applies the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process in the estimation of

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the volatility of the foreign exchange market in Kenya, based on daily exchange rates data from January, 1993 - December, 2006. The Kenya shilling was considered against the US dollar, Sterling pound, Japanese Yen, and Euro, because all official reserves and foreign currencies transaction in Kenya were held in these currencies. The GARCH process has the property of been heavy tailed, even when normality is assumed (Franke et al., 2008).

## ESTIMATION OF EXCHANGE RATE VOLATILITY

The Generalised Autoregressive Conditional Heteroscedastic process of order (p,q) introduced in Bollerslev (1986), as a generalisation of the Autoregressive Conditional Heteroscedastic process of order (p), abbreviated as ARCH(p), of Engle (1982), can be used to model the volatility in the exchange rate returns.

Let  $(Z_t)$  be the sequence of independent and identically

distributed (*iid*) random variables such that  $Z_t \sim N(0,1)$ . Then,  $\varepsilon_t$  is the  $GARCH(p, q)$  process if

$$\varepsilon_t = \sigma_t Z_t, t \in \mathbf{Z} \quad (1)$$

With

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2, t \in \mathbf{Z} \quad (2)$$

Where  $\sigma_t^2$  is a non-negative process, and  $\alpha_0 > 0$ ,  $\alpha_i \geq 0$  for  $i = 1, \dots, p$  while  $\beta_i \geq 0$  for  $i = 1, \dots, q$ . The non-negativity restrictions on the parameters ensure positivity of the variance  $\sigma_t^2$ .

The sizes of the parameters  $\alpha_i$  and  $\beta_i$  determine the short run dynamics of the resulting volatility process. Large ARCH error coefficients,  $\alpha_i$ , imply that volatility reacts significantly to market

movements. Large GARCH coefficients,  $\beta_i$ , indicate that shocks to the conditional variance take a long time to die out (that is volatility is persistent). High  $\alpha_i$  coefficient relative to  $\beta_i$  indicate that volatility tends to be more extreme.

Akaike Information Criteria (AIC) can be used to determine an appropriate lag length in the GARCH model. However, empirical evidence, according to Bollerslev et al. (1992), has shown that whilst relatively long lags are required in ARCH models, the GARCH (1,1) is usually adequate in describing many financial time series. This paper has adopted the latter one.

To obtain the quasi-likelihood function, we reformulated model (1) in the following way. Let the observed sequence  $(Y_t)$  follow the process

$$Y_t = C_0 + \varepsilon_{0t}, t = 1, 2, \dots, n$$

Where  $(\varepsilon_{0t})$  is assumed to be GARCH (1,1) process,  $\varepsilon_{0t} = \sigma_{0t} Z_t$  with  $(Z_t)$  being a sequence of *iid* random variables

and 
$$\sigma_{0t}^2 = \omega_0 (1 - \beta_0) + \alpha_0 \varepsilon_{0t-1}^2 + \beta_0 \sigma_{0t-1}^2, \quad (3)$$

$$\sigma_{0t}^2 = \omega_0 + \alpha_0 \sum_{k=0}^{\infty} \beta_0^k \varepsilon_{0t-1-k}^2, \text{ almost sure.}$$

Which is strictly stationary (Posedel, 2005), if  $E[\ln(\beta_0 + \alpha_0 Z^2)] < 0$  holds. We therefore assumed that the process was described with true parameters in the vector form given as  $\theta_0 = (C_0, \omega_0, \alpha_0, \beta_0)'$ . We also assumed the model of the form as

$$Y_t = C + \varepsilon_t, t = 1, 2, \dots, n \quad (4)$$

With unknown vector parameters given as  $\theta = (c, \omega, \alpha, \beta)'$  and  $\sigma_t^2(\theta) = \omega(1 - \beta) + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2(\theta)$ ,  $t = 2, 3, \dots, n$  with the initial condition  $\sigma_1^2(\theta) = \omega$ . The expression for the process of conditional variance can therefore be written as

$$\sigma_t^2 = \omega + \alpha \sum_{k=0}^{t-2} \beta^k \varepsilon_{t-1-k}^2 \quad (5)$$

Let the compact space,  $\Theta$ , be defined as

$$= \{ \theta : C_L \leq C \leq C_d ; 0 < \omega_L \leq \omega \leq \omega_d ; 0 < \alpha_L \leq \alpha \leq \alpha_d ; 0 < \beta_L \leq \beta \leq \beta_d < 1 \} \subseteq \{ \theta : E[\ln(\beta + \alpha Z^2)] < 0 \}$$

In addition, we assumed  $\theta_0 \in \Theta$ , be defined as  $\alpha_0 > 0$  and  $\beta_0 > 0$ . The quasi-likelihood function is then given as

$$L_T(\theta) = \frac{1}{2T} \sum_{t=1}^T L_t(\theta),$$

Where

$$L_t(\theta) = -\ln \sigma_t^2(\theta) - \frac{\varepsilon_t^2}{2\sigma_t^2(\theta)} \quad (6)$$

To avoid confusion with (5), let  $\sigma_{ut}^2(\theta) = \omega + \alpha \sum_{k=0}^{\infty} \beta^k \varepsilon_{t-1-k}^2$ , corresponding with  $\varepsilon_t = Y_t - C$ . to get

$$L_{uT}(\theta) = \frac{1}{2T} \sum_{t=1}^n L_{ut}(\theta),$$

where  $L_{ut}(\theta) = -\ln \sigma_{ut}^2(\theta) - \frac{\varepsilon_{ut}^2}{2\sigma_{ut}^2(\theta)}$

The vector of parameter value, denoted by  $\hat{\theta}_T$ , that maximize the likelihood function (6) on the set  $\Theta_1 \subset \Theta$  is obtained as:

$$\hat{\theta}_T = \underset{\theta \in \Theta_1}{\text{ARG MAX}} L_T(\theta)$$

The asymptotic properties of  $\hat{\theta}_T$  are stated under the following conditions:

(A1)  $(Z_t)$  is a sequence of iid random variables such that

$$E[Z_t] = 0;$$

(A2)  $Z_t^2$  is nondegenerate;

(A3) for some  $\delta > 0$ , there exists  $S\delta < \infty$  such that  $E[Z_t^{2+\delta}] \leq S < \infty$ ;

$$(A4) E[\ln(\beta_0 + \alpha_0 Z_t^2)] < 0;$$

(A5)  $\theta_0$  is in the interior of  $\Theta$ ;

(A6) If for some  $t$ ,  $\sigma_{0t}^2 = c_0 + \sum_{k=1}^{\infty} c_k \varepsilon_{t-k}^2$  and

$$\sigma_{0t}^2 = c_0^* + \sum_{k=1}^{\infty} c_k^* \varepsilon_{t-k}^2 \text{ holds, then } c_i = c_i^* \text{ for all } 1 \leq i < \infty;$$

(A7) The fourth moment of the random variable  $Z_t$  is finite.

The theorem, that gives the consistency and asymptotic normality properties of the parameter vector  $\hat{\theta}_T$ , is now stated.

### Theorem

Suppose conditions A1 - 7 holds, then

$$\sqrt{T} (\hat{\theta}_T - \theta_0) \rightarrow N(0, V_0)$$

$$\text{Where } V_0 = B^{-1} A B^{-1}, \text{ with } B_0 = B(\theta_0) = -E(\nabla^2 L_{ut}(\theta_0))$$

$$\text{and } A_0 = E(\nabla L_{ut}(\theta_0) \nabla L_{ut}(\theta_0)')$$

The proof of the theorem can be found in Francq and Zakoian (2004), where strong consistency and asymptotic normality of the quasi-likelihood estimator of GARCH process is given. See also Posedel (2005) for detailed demonstration on the asymptotic properties.

## EMPIRICAL RESULTS

The data used consists of daily exchange rates on the Kenya shillings against the US dollar, Euro, Sterling pound and Japanese Yen (100). The daily exchange rate

was an average of buying and selling rates of commercial banks spot exchange rates. The computation of the exchange rates is currently based on foreign exchange transactions of nine Kenyan commercial banks which are the major participants in the foreign exchange market. These are Barclays bank of Kenya, Citibank, Standard Chartered, Stanbic, Co-operative, National bank of Kenya, Bank of Africa, Commercial bank of Africa, and Kenya commercial bank.

Except for the Euro which was introduced in 1999, the rest of the exchange rates used in this paper cover the period 1993 - 2006. Consequently, the exchange rate of the Kenya shilling against the Euro comprises of 1997 observations while those against the US dollar, Sterling pound and Japanese Yen have 3501 observations each. The data were obtained from the Central Bank of Kenya database and are also published on a weekly basis in the Central Bank of Kenya's Weekly Bulletin of Economic Indicators at the Bank's website ([www.centralbank.go.ke](http://www.centralbank.go.ke)).

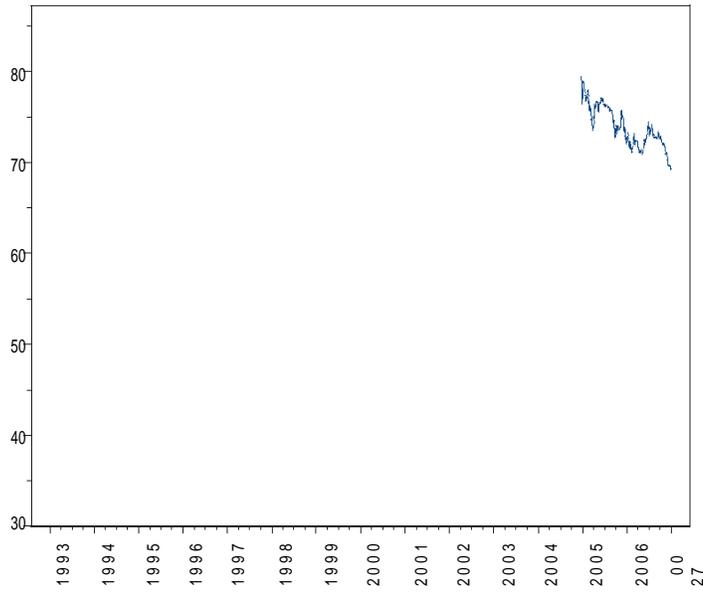
The choice of these currencies was based on their relative proportions in the Bank's foreign exchange investment portfolio, and their currency composition of imports. The currency composition of Kenya's imports comprised about 52% in US dollars in December 2006, CBK (2006). The foreign exchange reserves portfolio was held mainly in US dollars, Sterling pounds, Euros and Japanese Yens during the study period.

## Data exploration

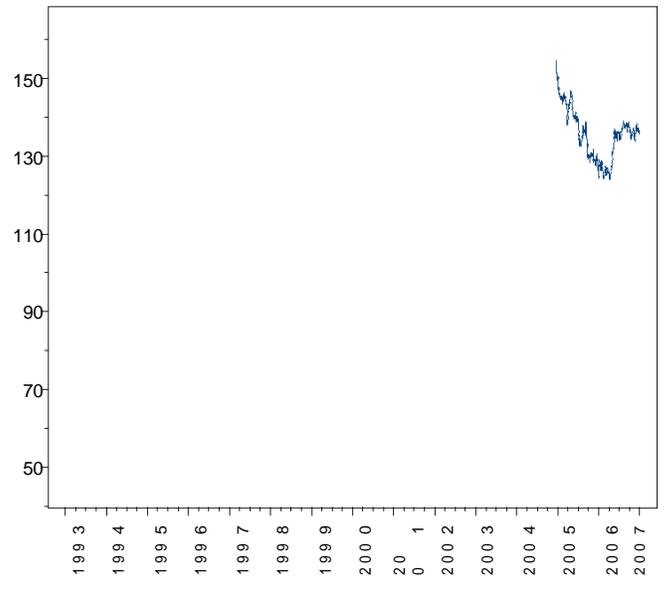
Plots in Figure 1 show general trends with high uncertainty in the exchange rate of the US dollar, Sterling pound and Japanese Yen between 1993 and 1998, and relative stability thereafter. Except for the year 2000 and 2003, the Euro was relatively stable compared with the other currencies. The strengthening of Kenya shilling between 1994 and 1995 was caused by the widened difference between Kenya shillings and foreign interest rates that attracted speculative capital inflows, elimination of foreign exchange licence in April, 1993 and the liberalization of offshore borrowing in May, 1994.

Exchange rate movements in the period could have also been affected by specific events and expectations. These include: failure by the government to implement specific expenditure management reforms as precondition for external aid disbursement from development partners; external aid inflows towards emergencies such as drought; significant private capital outflows; improved economic management; increased foreign exchange earnings from exports of goods and services; and capital inflows through remittances from Kenyans in the diaspora (CBK, 2006). In particular, the shilling depreciated sharply from 72.86 per US dollar on 6th October, 2004 to 79.47 per US dollar on 2nd December 2004 following an announcement by the International Monetary Fund (IMF) that it was delaying an expected disbursement pending

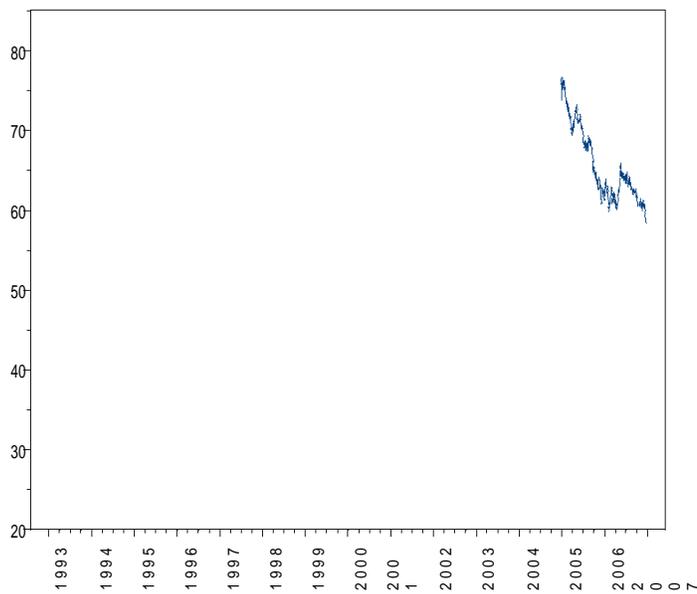
### Ksh/USD



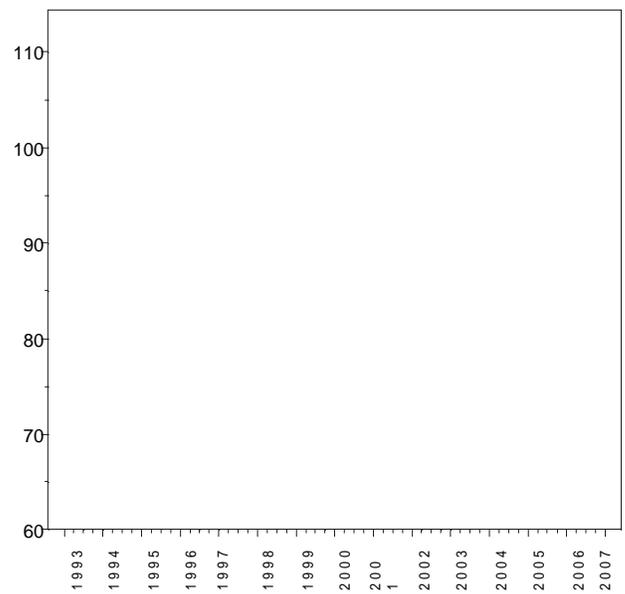
### Ksh/Pound



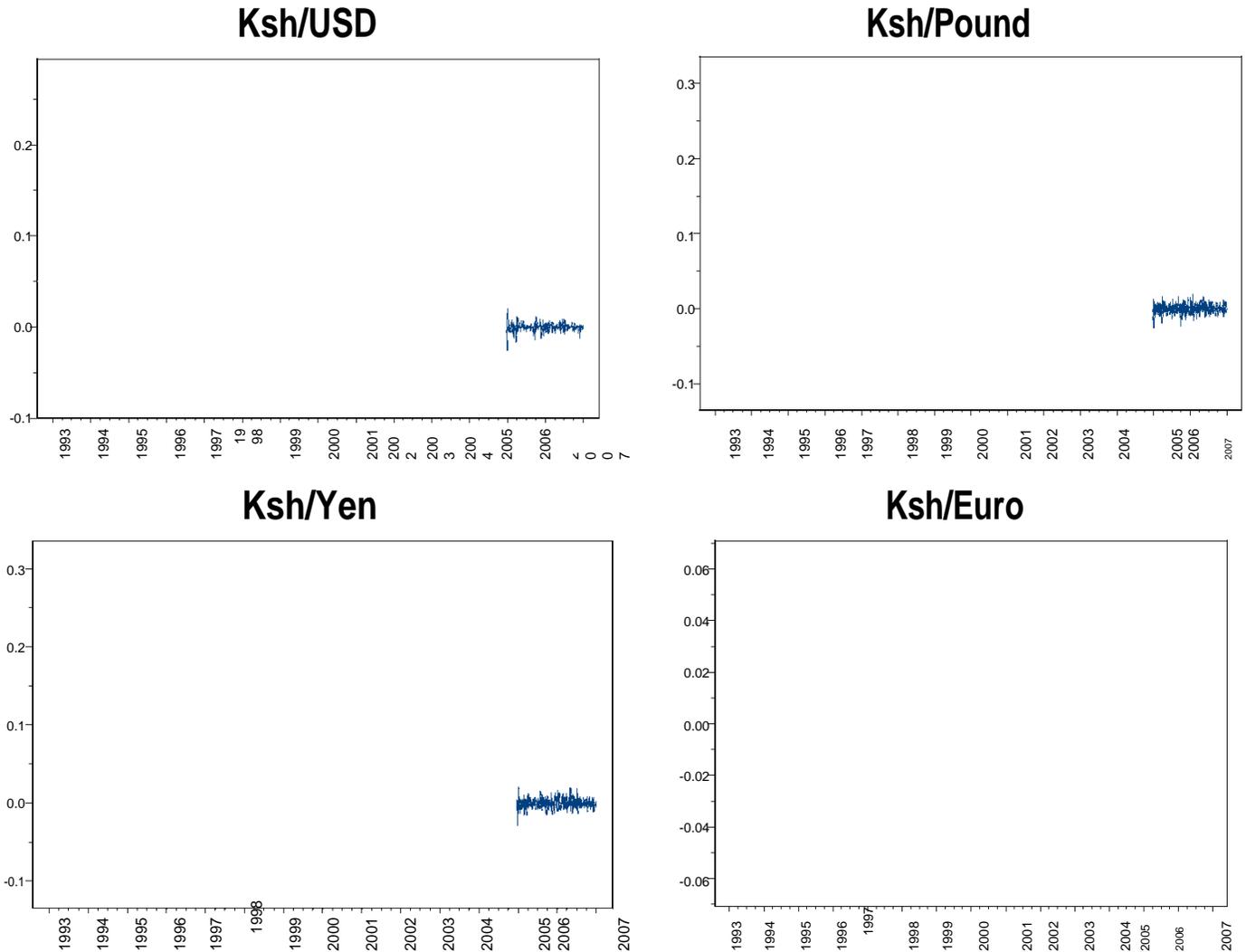
### Ksh/Yen



### Ksh/Euro



**Figure 1.** Trends in the exchange rate. Ksh/USD, Ksh/Pound, Ksh/Yen and Ksh/Euro refer to the exchange rate of the Kenya shilling to the US Dollar, Sterling Pound, Japanese Yen and Euro respectively. The Euro currency was introduced in 1999. Source: CBK



**Figure 2.** Exchange rate returns.

disbursement pending reforms on governance – Ministry of Finance (2006). This was interpreted by some players in the financial markets as an aid freeze and that other development partners would follow suit.

In order to estimate the volatility in the exchange rates, we used logarithm rates returns. The log-returns in plots in Figure 2 reveal dependence structure where periods of high returns tend to be followed by the high returns. This indicates volatility clustering in the data, the characteristics were consistent with empirical evidence in Embrechts et al. (1997) and Taylor (1986).

Descriptive statistics for the exchange rate returns are presented in Table 1. The mean of the log exchange rate returns range from 0.011% on the Euro to 0.026% on the Sterling Pound, which are negligible. The skewness coefficients were greater than zero. This indicates that the distributions of the exchange returns are not normal. The positive skewness coefficients indicate that the

distribution of the returns is slightly right skewed, implying that depreciations in the exchange rate occur slightly more often than appreciation. All the Kurtosis coefficients for log-returns of exchange rate of the Shilling against US dollar, Sterling pound, Euro and Japanese Yen are much greater than the three for a normal distribution. This indicates that the underlying distributions of the returns are leptokurtic. The Jarque-Bera test for normality indicates that the distribution of exchange rate returns for all the currencies have tails which are significantly heavier than that of the normal distribution.

### Estimated volatility models

We estimated the volatility in the four exchange rates using GARCH (1,1) model. The results of parameter estimates are shown in Table 2. The results show that the

**Table 1.** Summary statistics for exchange rate log-returns.

	Pound	Euro	USD	Japanese Yen
Mean	0.000260	0.000110	0.000185	0.000199
Median	-0.000040	-0.000010	-0.000040	-0.000140
Maximum	0.272460	0.056780	0.261380	0.276750
Minimum	-0.079400	-0.052380	-0.064780	-0.078230
Standard deviation	0.009868	0.007587	0.008342	0.010738
Skewness	0.4529640	0.039193	0.570640	0.414123
Kurtosis	255.364100	6.725157	461.849200	200.475400
Jarque-Bera statistic	9348233.0	1155.18	30861733.0	5729931.0
Jarque-Bera probability	0.000	0.000	0.000	0.000
No. of observations	3501	1997	3501	3501

Note: The Jarque and Bera (1987) test combines prior tests for skewness and kurtosis, and is used to test for normality of data. The test uses the statistic  $JB = \frac{S^2(X)}{(6/n)} + \frac{(K^2(X)-3)}{(24/n)} - \chi^2_{(2)}$ , where

$S(X) = \frac{(X - \alpha_X)^3}{\sigma_X^3}$  and  $K(X) = \frac{(X - \alpha_X)^4}{\sigma_X^4}$  are the skewness and kurtosis coefficients respectively.

**Table 2.** GARCH (1,1) models for exchange rates returns.

	Sample	$\omega$	$\alpha$	$\beta$	GED parameter
Ksh/USD	1993 - 2006	0.000000105 (5.899737)	0.096694 (10.92432)	0.890251 (34.36391)	0.545061 (61.42906)
	2000 - 2006	0.000000081 (3.836304)	0.137386 (7.118349)	0.790559 (32.72725)	0.640984 (30.15752)
	2003 - 2006	0.000000378 (3.493522)	0.086550 (4.610119)	0.800243 (19.74466)	0.693861 (18.71917)
Ksh/Euro	1993 - 2006	0.00000169 (2.992535)	0.057368 (5.631162)	0.912865 (53.4694)	1.550046 (27.89108)
	2000 - 2006	0.00000134 (3.046872)	0.058170 (5.84111)	0.917479 (62.58599)	1.586057 (26.05356)
	2003 - 2006	0.00000225 (1.97455)	0.072265 (4.37786)	0.885066 (26.54393)	1.624607 (18.09584)
Ksh/Pound	1993 - 2006	0.00000235 (7.955607)	0.124040 (10.02159)	0.844105 (147.8030)	1.004069 (81.52386)
	2000 - 2006	0.00000128 (2.733988)	0.070897 (5.339601)	0.897343 (44.2896)	1.427107 (26.38942)
	2003 - 2006	0.00000381 (2.18688)	0.076450 (4.183171)	0.836014 (16.56647)	1.554226 (16.46329)
Ksh/Yen	1993 - 2006	0.00000303 (10.44116)	0.090346 (8.567443)	0.874988 (140.6107)	1.024499 (94.81803)
	2000 - 2006	0.00000397 (3.128433)	0.078186 (4.7511)	0.832441 (21.94708)	1.45685 (22.75235)
	2003 - 2006	0.00000355 (2.551353)	0.079279 (4.088599)	0.839489 (19.90056)	1.438015 (17.93582)

Note: t-statistics are in parentheses.

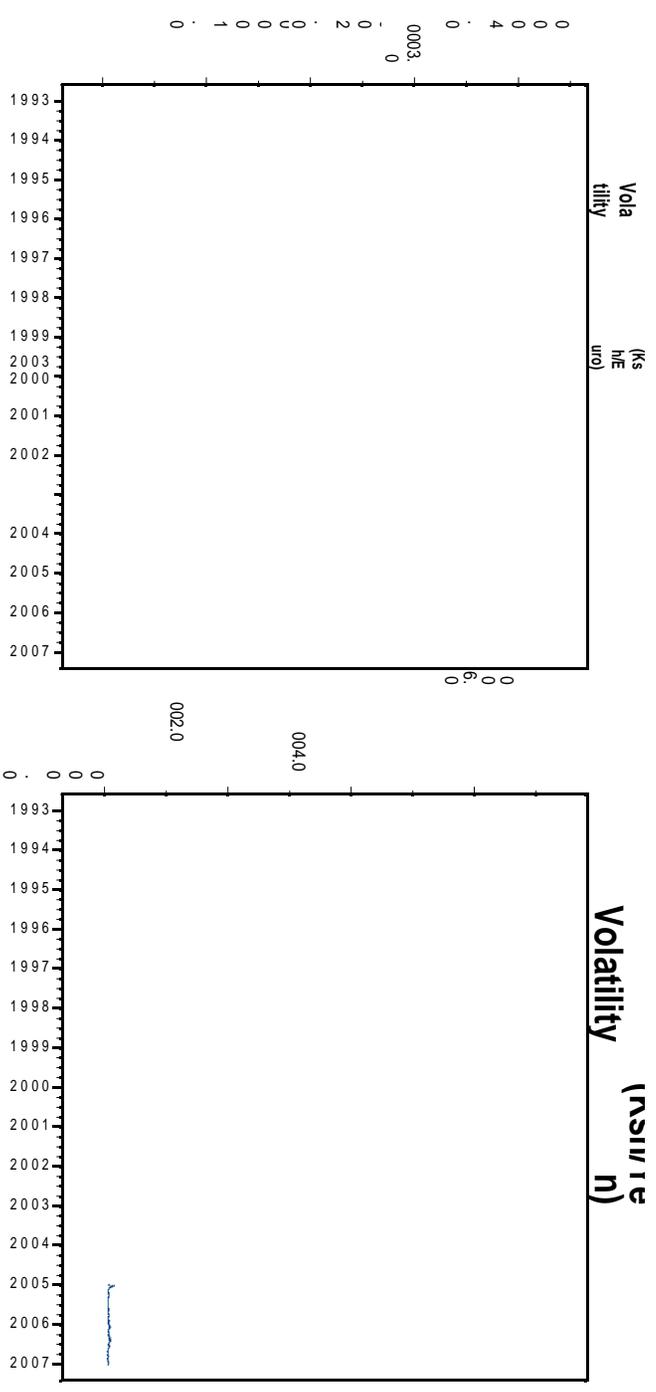
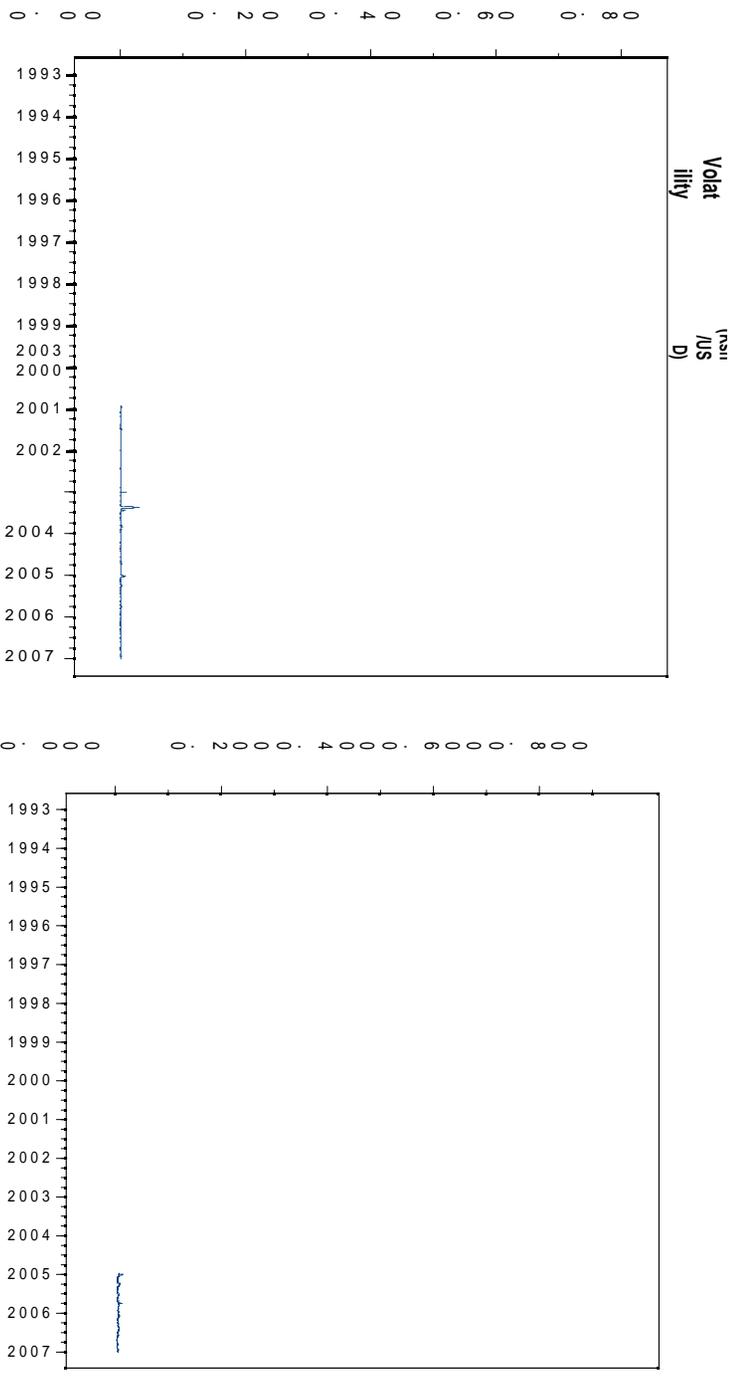


Figure 3. Exchange rate volatility.

estimated GARCH (1, 1) models are significant at 5% significance level. There is a high persistence of shocks in the volatility. The estimated  $\alpha$  and  $\beta$  parameters are positive on all currencies, and the sum is less than 1 for each of GARCH estimate. The QML estimates of the exchange rate returns on the US dollar (0.545), Euro (1.55), sterling pound (1.004) and Japanese Yen (1.024) are significant and correspond to distributions with heavier tails than the normal distribution.

The GARCH (1,1) models were re-estimated for sample periods 2000 - 2006 and 2003 - 2006 to examine the stability of the parameter estimates. The sum of the estimated  $\alpha$  and  $\beta$  parameters were less than one for all currencies. Furthermore, parameter estimates for all models did not change significantly using different samples. However, the sum of the parameters was almost unified for the shorter samples of returns on the US dollar, implying that the later could be a better fit.

Figure 3 shows plots of the estimated volatility process of the GARCH (1, 1) models in Table 2. The plots reveal decreasing volatility in the exchange rate returns implying relative stability in the exchange rate.

## Conclusion

GARCH (1,1) was applied in the estimation of volatility in the Kenyan foreign exchange market data for the period 1993 - 2006. Exploratory analysis showed that the exchange rates are leptokurtic and slightly positively skewed. This implies that the exchange rate depreciation was preferred during the period, probably to ensure that Kenya's exports remained competitive.

The Quasi-likelihood procedure used has parametric estimators that are consistent and asymptotically normal. The estimated models fit the data well, thereby confirming the empirical evidence in Bollerslev et al. (1992), that the GARCH (1,1) is adequate in describing volatility in many financial time series. Comparison of the three periods (1993 - 2006, 2000 - 2006 and 2003 - 2006) revealed some differences in the quality of fits. It may therefore be of interest to explore the concepts of change point detection to improve the overall fit for the whole period, 1993 - 2006.

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