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Full Length Research Paper

Multiple-level principal-agent model under adverse selection

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Almost all principal-agent models focus on single level situation, while there exist numerous cases of principal-agent relation with multiple levels in practice. This paper develops principal-agent models with multiple levels based on subcontract phenomena. The corresponding properties about principal-agent models with multiple levels under adverse selection are explored. There exists twist of the quantity in subcontract. We also find that the efficiency of principal-agent with multiple levels is lower than that with single level.

Key words: Principal-agent model, multiple levels, incentive mechanisms, industrial organization, game theory.

INTRODUCTION

The early incentive problem was ascended to labor divisions of the Adam Smith (Smith, 1776) and the theory of principal-agent has been developed for thirty years in economic field. Furthermore, principal-agent game plays crucial roles in industrial organization theory (Laffont and Martimort, 2002; Bolton and Dewatripont, 2005; Martimont, 1996; Martimont and Stole, 2002; Nie, 2009), which was extensively applied to bilateral trade (Myerson and Sattherwaite, 1983), public goods (Laffont and Martimort, 2005) and bargaining problem (Myerson, 1979). Principal-agent models are attached high importance by society (Maskin, 2008; Myerson, 2008; Tirole, 1998; Maskin and Tirole, 1990) and there exist numerous researches on principal-agent models. Actually, almost all extant principal-agent models focus on the single level situation. There exist many subcontract phenomena in society, which motivates us to extend classic principal-agent models to the situation with multiple levels. This paper characterizes subcontract phenomena. There popularly exist subcontracts in business field and in trading community. In a large project, for example, many subprojects are transferred to many companies according to technology necessity. When large scale equipment is required to make, many parts are subcontracted to other companies. There are some advantages for subcontract phenomena. Firstly, some difficulties in techniques can be smoothly settled by the subcontract with helps of other companies. Secondly, the efficiency can be improved by optimal allocation. Finally, some risk can be jointly undertaken by multiple companies. The subcontract seems extremely popular in Chinese highway projects in 1990s.

Certainly, there may exist some disadvantages in subcontracts. Firstly, subcontract may issues in corruptions, especially in the government. Many projects are directly or indirectly undertaken by government officials in charge of theses projects and these projects are then subcontracted to other companies while the corresponding officials earn much by these arbitrages. Secondly, under the incomplete information, the cheat behaviors frequently appear. Finally, the efficiency is reduced to a certain degree if the assignment of the resources is not good enough. All these motivate our research on subcontract phenomena both in theory and in the application community. We aim to establish principal-agent model for the subcontracts. We hope that our results can help to establish optimal subcontracts to eliminate the above potential disadvantages. The principal-agent game in this work is different from classic principal-agent models because there exist multiple contracts in the hierarchy in this system. On one hand, the model based on subcontracts is more difficult than classic principal-agent models. On the other hand, the efficiency of subcontract s seems more important than other contracts.

The main contribution of this paper in theory lies in the establishment of the principal-agent model with multiple levels and the theoretic analysis is correspondingly given. In application, we consider the extremely popular economic phenomena, the subcontract, in the society. These phenomena in economics are modeled and analyzed. We will establish principal-agent model with multiple levels under the incomplete information in this paper. The principal-agent model with the single level under the adverse selection was originally established by Mirrless (Mirrless, 1971) and was subsequently analyzed (Mussa and Rosen, 1978; Maskin and Riley, 1984). We hope to extend these adverse selection models to multiple levels in this work. This paper is organized as follows: The principal-agent model with multiple levels under adverse selection is outlined in Section 2. The corresponding properties are explored in Section 3. Some remarks are given in the final section.

THE MODEL

Here we formally give principal-agent models with multiple levels. In the following model, a principal and an agent in the upper level with a unique agent in the lower level are addressed. Furthermore, the agent in upper level simultaneously acts as the principal in lower level. The model is formally outlined as follows. For the upper level problem, $u_0(q)$ is the reservation utility at the upper level. $\theta \in \{\underline{\theta}, \overline{\theta}\}$ is the marginal cost to agent in upper level, where $\underline{\theta}$ relates to the higher efficiency with probability $\lambda \in [0,1]$ and $\overline{\theta}$ to the lower efficiency with probability $1-\lambda$. Furthermore, $\underline{\theta} > \overline{\theta}$. $T \in \{\underline{T}, \overline{T}\}$ is the corresponding transfer from the principal to the agent in upper level. $S(q)$ is the utility function of the principal along with the corresponding quantity q. Furthermore, to simplify the problem, the linear cost function is always employed in this paper. We therefore have the following upper problem.

$$\underset{\underline{q},\overline{q},\underline{T},\overline{T}}{Max} \quad \lambda[S(\underline{q}) - \underline{T}] + (1-\lambda)[S(\overline{q}) - \overline{T}]. \tag{1}$$

$$S.T. \quad \underline{T} - \underline{\theta}\,\underline{q} \ge u_0(\underline{q}), \tag{2}$$

$$\overline{T} - \overline{\theta}\,\overline{q} \ge u_0(\overline{q}), \tag{3}$$

$$\overline{T} - \overline{\theta}\,\overline{q} \ge \underline{T} - \overline{\theta}\,\underline{q}, \tag{4}$$

$$\underline{T} - \underline{\theta}\,\underline{q} \ge \overline{T} - \underline{\theta}\,\overline{q}. \tag{5}$$

For the above principal-agent game in the upper level, Equations (2) and (3) are participant constraints and Equations (4) and (5) are incentive constraints. For lower level principal-agent problem, the agent in upper level acts as the principal. The reservation utility is zero for the agent in lower level. $\theta^L \in \{\underline{\theta}^L, \overline{\theta}^L\}$ is the marginal cost to the agent in lower level, where $\underline{\theta}^L$ relates to the higher efficiency with probability $\omega \in [0,1]$ and $\overline{\theta}^L$ to the lower efficiency with probability $1-\omega$. Furthermore, $\underline{\theta}^L < \overline{\theta}^L$. $t \in \{\underline{t}, \overline{t}\}$ is the corresponding transfer from principal to agent in lower level. The lower level model with principal-agent is therefore given as follows:

$$\underset{\underline{t},\overline{t},\underline{q}^L,\overline{q}^L}{Max} \quad \omega[u_0(\underline{q}^L) - \underline{t}] + (1-\omega)[u_0(\overline{q}^L) - \overline{t}]. \tag{6}$$

$$S.T. \quad \underline{t} - \underline{\theta}^L \underline{q}^L \ge 0, \tag{7}$$

$$\overline{t} - \overline{\theta}^L \overline{q}^L \ge 0, \tag{8}$$

$$\overline{t} - \overline{\theta}^L \overline{q}^L \ge \underline{t} - \overline{\theta}^L \underline{q}^L, \tag{9}$$

$$\underline{t} - \underline{\theta}^L \underline{q}^L \ge \overline{t} - \underline{\theta}^L \overline{q}^L. \tag{10}$$

$$\omega \underline{q}^L + (1-\omega)\overline{q}^L = \lambda \underline{q} + (1-\lambda)\overline{q}. \tag{11}$$

In the above principal-agent game in the lower level, equations (7) and (8) are the participant constraints and equations (9) and (10) are the incentive constraints. We point out that the reservation utility at upper level $u_0(q)$ is exogenously determined by lower level problem. Equation (11) is market clearance condition to simplify the model. (If equation (11) does not hold, we can discuss the model similarly but it is more difficult to handle). The expected productions of the upper principal are exactly consistent with those of the principal at lower level. The above system with equations (1) to (11) constitutes a principal-agent game with two levels. In this system, the agent in upper level simultaneously acts as the principal in the lower level. When the corresponding player makes decisions, he/she should fully consider the principal in upper level and the agent in lower level. This issue is the difficulty for the principal-agent model with two levels.

To simplify the problem, we consider the situation with the unique principal and the unique agent both in the upper level and in the lower level. When there exist multiple principals or multiple agents, the problem becomes more complicated. Furthermore, without loss of generality, the fixed cost is always assumed to be zero to simplify the problem. The timing of the game is given as follows: Firstly, the principal proposes the contract to the agent in upper level. Secondly, the principal correspondingly presents the subcontract to the agent in lower level. All the agents are not mandatory. Thirdly, the agent in lower level accepts or rejects the subcontract. Finally, the agent in the upper level subsequently accepts or rejects the contract (When the agent in lower level rejects, the agent in upper level correspondingly rejects. When the agent in the lower level accepts, the agent in the upper level correspondingly reject accepts) and the game is over. There is research on game theory with multiple levels in applied mathematics (Nie, 2007; Nie, 2009). The general way to handle this problem is to transform it into single level problem. The optimization methods with single level are then employed to tackle these problems. For convenience, the following assumption is given to guarantee the existence and the unique of the solution for above system. Assumption (A) $u_0(q)$ is continuous and satisfies $u_0(0)=0$, $u_0'(q)>0$ and $u_0''(q)<0$ for all q. (B) $S(q)$ is also continuous and satisfies $S(0)=0$, $S'(q)>0$ and $S''(q)<0$ for all q. (C) $(1-\lambda)S(q) - u_0(q)$ also satisfies $S(0) - u_0(0) = 0$, $(1-\lambda)S'(q) - u_0'(q) > 0$ and $(1-\lambda)S''(q) - u_0''(q) < 0$ for all q. (A) guarantees the existence and the unique of the solution to lower level problem. (C) guarantees the existence and the unique solution to upper level problem. By the way, (B) implies the concave of $S(q)$. (C) means that the preference of the principal is much more than that of the agent in the upper level.

RESULTS

The properties of the above model are discussed. For the principal and the corresponding agent both in the upper and lower levels, the properties are considered thus.

The principal and the agent in the upper and in the lower level

For the upper level problem and the lower level problem, it has been extensively considered in the papers about incentive theory. Combined the above problem, we give the solutions to the upper level and the lower level, respectively. For the upper level problem, the convex property of the feasible set is met if the agent is risk-neutral. In general, for the concave function $u_0(q)$, the property of the convex is destroyed because the set $S_1 = \{(T, \underline{q}) \mid \underline{T} - \underline{\theta}\underline{q} \geq u_0(\underline{q})\}$ is not convex, which issues in the difficulties to handle the upper principal- agent problem (Rockafellar, 1970). The hypothesis (C) guarantees the existence of the solution to the upper level problem. If the above assumption is satisfied, the following result is obtained according to the classic result about incentive theory with hidden information (Laffont and Martimont, 2002; Bolton and Dewatripont, 2005). Because the reservation utility is $u_0(q)$, the problem seems more difficult than that in (Laffont and Martimont, 2002; Bolton and Dewatripont, 2005):

Proposition 1

For the optimal contract of the upper level, $\underline{q}^* \leq \bar{q}^*$. Furthermore, equations (3) and (5) are all binding. Equations (2) and (4) can be induced by equations (3) and (5). Furthermore, $\underline{T} - \underline{\theta}\underline{q} \geq u_0(\underline{q})$ if $\underline{q}$ >0. The agent with the higher efficiency obtains the second -best optimal solution, which is just the optimal solution. Namely,

$$S'(\underline{q}) = \underline{\theta}. \quad (12)$$

The utility of the agent with the lower efficiency obtains the second-best optimal solution, which is lower than the optimal solution. Namely;

$$S'(\bar{q}^*) = \bar{\theta} - \frac{(\underline{\theta} - \bar{\theta})\lambda - u_0'(\bar{q}^*)}{1-\lambda} \quad . \quad (13)$$

Proof: For the optimal contract of the upper level, from Equations (4) and (5), we have $(\underline{T} - \underline{\theta}\underline{q}) - (\bar{T} - \underline{\theta}\bar{q}) \geq (\underline{T} - \bar{\theta}\underline{q}) - (\bar{T} - \bar{\theta}\bar{q})$. Namely, $\underline{\theta}\bar{q} - \bar{\theta}\bar{q} \geq \underline{\theta}\underline{q} - \bar{\theta}\underline{q}$. According to $\bar{\theta} \geq \underline{\theta}$, we have $\underline{q}^* \leq \bar{q}^*$ and $u_0(\underline{q}^*) \leq u_0(\bar{q}^*)$. We show Equation (2) based on Equations (3) and (5). From Equation (5), we have $\underline{T} - \underline{\theta}\underline{q} \geq \bar{T} - \underline{\theta}\bar{q} \geq \bar{T} - \bar{\theta}\bar{q} \geq u_0(\bar{q}) \geq u_0(\underline{q})$. Namely, Equation (2) is induced by Equations (3) and (5). If $\underline{q}$ >0, from $\underline{q} \leq \bar{q}$, we have $\bar{q}$ >0. Thus, we obtain $\underline{T} - \underline{\theta}\underline{q} > u_0(\underline{q})$ according to the above proof. Here, we prove that Equation (3) is binding by contradiction. If equation (3) is not binding, the principal can simultaneously reduce $\underline{T}$ and $\bar{T}$ such that equation (3) is binding along with Equations (4) and (5) all holding. The profits of the principal are improved. This contradicts the optimal contrary. Namely, Equation (3) is binding at the optimal solution. Similarly, we can show that equation (5) is also binding by contradiction. If Equation (5) were not binding, we should have $\underline{T} - \underline{\theta}\underline{q} \geq \bar{T} - \underline{\theta}\bar{q}$. The principal can reduce the transfer $\underline{T}$ to improve the profits and Equations (2) to (4) all hold. This contradicts the hypothesis of the optimal contract.

Furthermore, Equation (4) can be induced by Equations (3) and (5). Because Equation (5) is binding, we have $\bar{T} - \underline{T} = \underline{\theta}(\bar{q} - \underline{q}) \geq \bar{\theta}(\bar{q} - \underline{q})$.

Namely, $\bar{T} - \bar{\theta}\bar{q} \geq \underline{T} - \bar{\theta}\underline{q}$ and equation (4) is immediately obtained. Here we show equations (12) and (13). Combined equations (3), (5) and (1), we have the following maximization problem.

$$\max_{\underline{q}, \bar{q}} \lambda[S(\underline{q}) - \bar{\theta}\bar{q} + \underline{\theta}\bar{q} - \underline{\theta}\underline{q} - u_0(\bar{q})] + (1-\lambda)[S(\bar{q}) - \bar{\theta}\bar{q} - u_0(\bar{q})].$$

Correspondingly considering the first order optimal conditions to the above unconstrained problem, we immediately obtain $S'(\underline{q}) = \underline{\theta}$ and $S'(\bar{q}^*) = \bar{\theta} - \frac{(\underline{\theta} - \bar{\theta})\lambda - u_0'(\bar{q}^*)}{1-\lambda}$, which are just Equations (12) and (13). Equation (12) is exactly equal to the output under the monopoly or the optimal solution is obtained for the higher efficiency agent. The result is therefore obtained and the proof is complete. The above result shows the twist of the products for the lower efficiency agent in the upper level, which is consistent with the results in the incentive theory. The solution to the upper level is determined by Equations (3), (5), (12) and (13). For the lower level problem, the constraint equation (11) has some effects on the solution to the system. The following result is obtained according to the classic result about the incentive theory with the adverse selection. Equations (8) and (10) are binding, while Equations (7) and (9) are induced by Equations (8) and (10). Equations (6) to (11) are equivalent to the following problem.

$$\max_{\underline{q}^L, \bar{q}^L} \omega[u_0(\underline{q}^L) - \underline{\theta}^L\underline{q}^L + \underline{\theta}^L\bar{q}^L - \bar{\theta}^L\bar{q}^L] + (1-\omega)[u_0(\bar{q}^L) - \bar{\theta}^L\bar{q}^L]. \quad (14)$$

(14)

$$S.T.\quad \omega \underline{q}^L + (1-\omega)\overline{q}^L = \lambda \underline{q} + (1-\lambda)\overline{q}.$$

Considering the first order optimal conditions to Equation (14) and letting μ is the corresponding Lagrangian multiplier for Equation (14), we immediately obtained

$$\omega[u_0'(\underline{q}^L) - \underline{\theta}^L + \mu] = 0, \qquad (15)$$

$$\omega[\underline{\theta}_L - \overline{\theta}^L] + (1-\omega)[u_0'(\overline{q}^L) - \overline{\theta}^L + \mu] = 0. \quad (16)$$

or

$$\omega[\underline{\theta}_L - \overline{\theta}^L] + (1-\omega)[u_0'(\overline{q}^L) - \overline{\theta}^L] = (1-\omega)[u_0'(\underline{q}^L) - \underline{\theta}^L] \quad (17)$$

Similar to the proof in Proposition 1, the corresponding results for the lower level principal-agent model are given as follows and the detail proof is omitted.

Proposition 2

For the optimal contract of the lower level, Equations (8) and (10) are all binding. Equation (7) and (9) can be induced by Equations (8) and (10). Furthermore, the agent with the higher efficiency obtains the second-best optimal solution, which is just the optimal solution under the product constraint equation (11). The agent with the lower efficiency obtains the second-best optimal solution, which is lower than the optimal solution. The information rent for the higher efficiency agent is strictly greater than zero. In brief, for the principal-agent problem with multiple levels, there exists twist in the products both for the higher efficiency agent and for the lower efficiency agent. The above result shows that the twist of the products for the lower efficiency agent under the quantity constraint Equation (11), which is consistent with the results in the incentive theory. On the other hand, the quantity constraint twists the solution to the higher efficiency agent. The solution to the lower level is determined by Equations
(8), (10), (11) and (17). According to the analysis, the optimal contract with two levels is uniquely determined by the following system of equation.

$$\overline{T} - \overline{\theta}\overline{q} = u_0(\overline{q}),$$
$$\underline{T} - \underline{\theta}\underline{q} = \overline{T} - \underline{\theta}\overline{q},$$
$$S'(\underline{q}) = \underline{\theta},$$
$$S'(\overline{q}) = \overline{\theta} - \frac{(\underline{\theta} - \overline{\theta})\lambda - u_0'(\overline{q})}{1-\lambda},$$
$$\overline{t} - \overline{\theta}_L \overline{q}_L = 0,$$
$$\underline{t} - \underline{\theta}^L \underline{q}^L = \overline{t} - \underline{\theta}_L \overline{q}^L, \qquad (18)$$

$$\underline{q}^L + \overline{q}^L = \underline{q} + \overline{q},$$
$$\omega[\underline{\theta}^L - \overline{\theta}^L] + (1-\omega)[\overline{u}_0'(q^L) - \theta^L] = \underline{\omega}[u_0'(\underline{q}^L) - \theta^L].$$

We here summarize the above results as follows. The optimal contract with two levels has the following properties:

(i) The higher efficiency agent in the upper level obtains the optimal profits while the lower efficiency one obtains the second-best optimal profits.
(ii) The products of the agent in the lower level are twisted because of the constraint (11), which is different from the results without the quantity constraint.
(iii) The principal-agent models with multiple levels are more difficult than those under single level.
(iv) From $\underline{T} - \underline{\theta}\underline{q} > u_0(\underline{q})$, we know that the higher efficiency agent obtains positive profits from the principal-agent game with two levels.

The optimal contract

We here analyze the optimal solution to the principal-agent with two levels in this subsection. Namely, the equilibrium conditions or the system of Equations (18) are considered in this subsection. By the implicit function theorem of comparative static analysis strategy, we consider Equations (18) and the following results hold.

Proposition 3

For the optimal contract, we have the following conclusion: When the marginal costs of the upper level increases, the quantity of the products of the corresponding agent decreases. Namely, $\frac{\partial \overline{q}^*}{\partial \overline{\theta}} < 0$ and $\frac{\partial q^*}{\partial \theta} < 0$ all hold. Furthermore, the quantity of the higher efficiency agent in the upper level has no relation with the parameter $\overline{\theta}$ and λ. Namely, $\frac{\partial \underline{q}^*}{\partial \overline{\theta}} = 0$ and $\frac{\partial \underline{q}^*}{\partial \lambda} = 0$. For the lower efficiency agent in the upper level, we have $\frac{\partial \overline{q}^*}{\partial \underline{\theta}} > 0$ and $\frac{\partial \overline{q}^*}{\partial \lambda} > 0$.

Proof:

For the third equation of (18), we rewrite it as $f_3 = S'(\underline{q}) - \underline{\theta} = 0$. We further have $\frac{\partial f_3}{\partial \underline{q}} = S''(\underline{q}) < 0$ and $\frac{\partial f_3}{\partial \underline{\theta}} = -1 < 0$. According to the implicit function theorem, there exists the unique solution $\underline{q}(\underline{\theta})$, which is also differentiable in θ. By the differential of the equation $f_3 = S'(\underline{q}) - \underline{\theta} = 0$, we have

$$\frac{\partial \underline{q}}{\partial \underline{\theta}} = -\frac{\frac{\partial f_3}{\partial \underline{\theta}}}{\frac{\partial f_3}{\partial \underline{q}}} < 0 .$$

$\frac{\partial \bar{q}^*}{\partial \bar{\theta}} < 0$ is therefore obtained. By the similar way, considering the fourth equation of (18), let $f_4 = S'(\bar{q}) - \bar{\theta} + \frac{(\underline{\theta} - \bar{\theta})\lambda - u'(\bar{q})}{1-\lambda} = 0$. Similar to the way to analyze f_3, we have $\frac{\partial \bar{q}^*}{\partial \bar{\theta}} < 0$.

According to the third Equation of (18), we immediately have $\frac{\partial \underline{q}^*}{\partial \bar{\theta}} = 0$ and $\frac{\partial \underline{q}^*}{\partial \lambda} = 0$. That is, the quantity of the higher efficiency agent has no relation with the parameter $\bar{\theta}$ and λ. Considering

$$f_4 = S'(\bar{q}) - \bar{\theta} + \frac{(\underline{\theta} - \bar{\theta})\lambda - u'(\bar{q})}{1-\lambda} = 0 \quad \text{or}$$

$$g_4 = (1-\lambda)[S'(\bar{q}) - \bar{\theta}] + (\underline{\theta} - \bar{\theta})\lambda - u'_0(\bar{q}) = 0,$$

we have $\frac{\partial g_4}{\partial \bar{q}} < 0$ from (C) of assumption, $\frac{\partial g_4}{\partial \underline{\theta}} > 0$ and $\frac{\partial g_4}{\partial \lambda} = \underline{\theta} - S'(\bar{q}) > 0$. $\frac{\partial g_4}{\partial \lambda} = \underline{\theta} - S'(\bar{q}) > 0$ is obtained from $\underline{q}^* \leq \bar{q}^*$, $S'(\underline{q}) = \underline{\theta}$ and the concavity of $S(q)$. We thus have $\frac{\partial \bar{q}}{\partial \underline{\theta}} > 0$ and $\frac{\partial \bar{q}^*}{\partial \lambda} > 0$. The improvement of the efficiency or the probability with the higher efficiency agent causes the increase of the quantity of the products of the lower efficiency agent. The result is therefore obtained and this completes the proof. Considering the principal-agent under the lower level, we have the similar result by the way similar to the above proposition.

Proposition 4

For the optimal contract in this paper, when the marginal costs of the lower level increase, the quantity of the products for the corresponding agent decreases. Namely, $\frac{\partial \bar{q}^{*,L}}{\partial \bar{\theta}_L} < 0$ and $\frac{\partial \underline{q}^{*,L}}{\partial \underline{\theta}_L} < 0$ all hold.

The proof of this result is similar to proposition 3 and the detail proof is omitted. This result illustrates that the higher (lower) marginal costs may result in the less (larger) quantity of the corresponding products. We here summarize the main results in this subsection as follows. The optimal contract with two levels has the following properties.

(i) The quantity of the agent in the upper level decreases (increases) if the marginal costs increase (decrease). Furthermore, the optimal quantity of the higher efficiency agent in the upper level has no relation with the parameter $\bar{\theta}$ and λ. For the lower efficiency agent in the upper level, we have $\frac{\partial \bar{q}^*}{\partial \underline{\theta}} > 0$ and $\frac{\partial \bar{q}^*}{\partial \lambda} > 0$.

(ii) The quantity of the agent in the lower level decreases (increases) if the marginal cost increases (decrease).

Compare with the benchmark

We here compare the subcontract with the contract under the single level. If no subcontract appears, it is the classic principal-agent model for the single contract, which is stated as follows.

$$\max_{\underline{q}, \bar{q}, \underline{T}, \bar{T}} \lambda [S(\underline{q}) - \underline{T}] + (1-\lambda)[S(\bar{q}) - \bar{T}]. \quad (19)$$

$$S.T. \; \underline{T} - \underline{\theta}\underline{q} \geq 0,$$
$$\bar{T} - \bar{\theta}\bar{q} \geq 0,$$
$$\bar{T} - \bar{\theta}\bar{q} \geq \underline{T} - \bar{\theta}\underline{q},$$
$$\underline{T} - \underline{\theta}\underline{q} \geq \bar{T} - \underline{\theta}\bar{q}.$$

The solution to the above principal-agent game with single level is given by the following system of the equations.

$$\bar{T} - \bar{\theta}\bar{q} = 0,$$
$$\underline{T} - \underline{\theta}\underline{q} = \bar{T} - \underline{\theta}\bar{q},$$
$$S'(\underline{q}) = \underline{\theta}, \quad (20)$$
$$S'(\bar{q}) = \bar{\theta} - \frac{\lambda}{1-\lambda}(\underline{\theta} - \bar{\theta}).$$

Let the optimal solution to the above system of the equations be $(\underline{q}^s, \bar{q}^s, \underline{T}^s, \bar{T}^s)$ and the optimal expected value (or the optimal expected utility) be V^S for the principal. We also assume the optimal solution to Equation (18) to be $(\underline{q}^*, \bar{q}^*, \underline{T}^*, \bar{T}^*)$ and the optimal expected value (or the optimal expected utility) to be V^* for the principal in the upper level. Comparing Equations (18) and (20), we have $S'(\underline{q}^s) = S'(\underline{q}^*) = \underline{\theta}$ and $S(\underline{q}^s) = S(\underline{q}^*)$ is obtained. We further have $\underline{q}^s = \underline{q}^*$ and

$\bar{q}^s < \bar{q}^*$ according to the monotonously increasing function $S(q)$. From equations (18) and the above system of equations, we further have

$$\underline{T}^s = \bar{\theta}\bar{q}^s - \bar{\theta}\underline{q}^s + \underline{\theta}\underline{q}^s = \underline{\theta}\underline{q}^s - (\bar{\theta}-\underline{\theta})\, \underline{q}^s,$$
$$\underline{T}^* = \bar{\theta}\bar{q}^* - \underline{\theta}\bar{q}^* + \underline{\theta}\underline{q}^* + u_0(\bar{q}^*) = \underline{\theta}\underline{q}^* - (\underline{\theta}-\bar{\theta})\, \bar{q}^* + u_0(\bar{q}^*).$$

Namely, from $\underline{q}^s = \underline{q}^*$ and $\bar{q}^s < \bar{q}^*$, we have $\bar{T}^s > \bar{T}^* - u_0(\bar{q}^*)$. Comparing the objective function of Equations (1) to (5) with the above principal-agent game with single level, we have the following conclusion.

Proposition 5

$$V^s \geq V^*.$$

Proof

Denote the feasible set to the above principal-agent with the single level to be

$$\Phi^S = \{\underline{T}, \bar{T}, \underline{q}, \bar{q}, \underline{q} \mid \bar{T} - \bar{\theta}\bar{q} \geq 0,\ \underline{T} - \underline{\theta}\underline{q} \geq \bar{T} - \underline{\theta}\bar{q}\}.$$

Similarly, the feasible set of the principal-agent model with the two levels is assumed to be

$$\Phi^* = \{\underline{T}, \bar{T}, \underline{q}, \bar{q} \mid \bar{T} - \bar{\theta}\bar{q} \geq u_0(\bar{q}),\ \underline{T} - \underline{\theta}\underline{q} \geq \bar{T} - \underline{\theta}\bar{q}\}.$$

It is obvious that the two sets satisfy the relation $\Phi^* \subseteq \Phi^S$. According to these two optimization problems, the objective functions are the same while the feasible sets are different. We immediately obtain that

$$V^* = \underset{\underline{q}, \bar{q}, \underline{T}, \bar{T} \in \Phi^*}{Max}\ \lambda[S(\underline{q}) - \underline{T}] + (1-\lambda)[S(\bar{q}) - \bar{T}]$$
$$\leq \underset{\underline{q}, \bar{q}, \underline{T}, \bar{T} \in \Phi^S}{Max}\ \lambda[S(\underline{q}) - \underline{T}] + (1-\lambda)[S(\bar{q}) - \bar{T}] = V^s.$$

The result $V^s \geq V^*$ is therefore obtained and the proof is complete.

The expected utility of the principal with the single level is no less than that of the principal with two levels. In the other words, the subcontract reduces the efficiency of the system. Thus, arbitrage may appear in the principal-agent with two levels in many situations under adverse selection for the agent with higher efficiency. When the costs are considered or the costs are far greater than zero, there may exist no feasible points in reality for the principal-agent model with the single level. For example, there is a large project required to invest $F + f$, which can not be undertaken by a company. There exists no solution for the principal-agent problem with the single level while there is at least a solution for the principal-agent problem with two levels if the principal and the agent in the lower level undertake F and f, respectively. Here, we summarize the results in this subsection as follows. Comparing with the classic principal-agent game under the single level, the optimal contract with two levels has the following properties.

(i) The principal-agent with two levels has lower efficiency than that with the single level under the same conditions.
(ii) When the costs are considered, the high costs can be undertaken by multiple companies for the principal-agent with two levels. This avoids that the higher costs result in the infeasibility of the contract under the single level. This result is very rational because there exist subcontracts in many large projects in the reality.

CONCLUSION

In this work, based on some economic phenomena, the theory of principal-agent model is extended to the situation under two levels. The theoretic results are analyzed. In the applications, the phenomena of subcontract are rationally considered. The principal-agent model with multi-level seems much more complicated than the classic principal-agent model. There exist some further researching topics about principal -agent model with multiple levels. On one hand, the existence of the solution to the systems is interesting. On the other hand, the applications about this model seem exceedingly important in the society. The principal-agent game with tree or more levels is also an interesting topic.

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