

*Full Length Research Paper*

# Optimal marketing and production planning with reliability consideration

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One of the primary assumptions on many optimal pricing strategies is that there is no imperfect product. This simple assumption often makes them impractical to use, since it is almost impossible to manage a production with virtually no defect. In this paper, the study proposes a new method where the reliability of the production is incorporated into pricing, marketing and production planning. The integrated model of this paper simultaneously determines price of products, marketing expenditure, lot size, setup cost, inventory holding cost and reliability of the production process. This model is formulated as a nonlinear optimization problem and the optimal solution in closed form is derived using geometric programming. In order to examine the behaviour of the proposed method, the study tests the modeling formulation using a numerical example.

**Key words:** Optimal pricing, marketing planning, production management, geometric programming.

## INTRODUCTION

In today's highly competitive environment, marketing and production decisions are among the most critical management concerns. Traditionally, firms have tended to treat these crucial decisions separately. However, today managers are well aware that coordinating marketing and production strategies reduces their conflicts and can provide a remarkable advantage to companies. Similarly, it has been proven that the coordination of price decisions with other aspects of the companies such as production, marketing and inventory management is not only helpful, but is essential (Chan et al., 2004). The purpose of this paper is to provide a framework for integrating pricing, marketing and production decisions and optimize the system rather than individual elements. The pricing issue has been the focus of numerous researchers. Fathian et al. (2009) study the pricing for electronic devices which is supposed to be sold on the internet. They found an optimal solution for their problem formulation and the solution is analyzed when the models' parameters are changed. Che (2009) develops a pricing strategy and reserved capacity plan based on product life cycle and production function on

television manufacturer. In his assignment, meta-heuristic techniques are used to determine the near optimal solution for a relatively highly nonlinear model.

Parlar and Weng (2007) proposed a coordinating pricing and production decisions in the presence of price competition. They develop a method based on Geometric Programming (GP) to find the optimal solution. Safaei et al. (2006) considered a price discrimination model when a product is sold in two different states under different conditions. Sadjadi and Ziaee (2006) also introduced a multi-objective decision making technique for a price discrimination problem and, using lexicography method, determined the optimal price in two phases. The multi-objective technique is also used for marketing planning and lot sizing by Islam (2008). Liu (2006) develops a computational method for the profit bounds of inventory model with interval demand and unit cost. Liu (2007) considers a discount model in his profit maximization problem and provides the optimal solution using GP. The model is also formulated in a form of nonlinear and non-convex function and the near optimal solution is determined using genetic algorithm.

Lee and Kim (1993) are believed to be among the first scholars who investigated optimal pricing and marketing expenditure using an integrated model. Their models determine the optimal pricing and marketing along with

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the lot-sizing, utilizing a mathematical modeling formulation with an adaptation of the GP. Lee and Lee (1999) propose a distributed decision support system approach to coordinating production and marketing decisions where the model could train itself by updating the information over a time horizon. Lee et al. (1996) introduced an optimal demand rate, lot sizing, and process reliability improvement decisions. They explain the effects of reliability on lot-sizing. Sadjadi et al. (2005) extended Lee and Kim's (1993) formulation for more realistic problems where demand and production rates have close relationships. Sadjadi et al. (2003) studied the optimal pricing models when research and development expenditures are involved in production. Jung and Klein (2006) compared three models of joint pricing and lot sizing with variable unit production cost through GP. Likewise, Lee (1993), Kim and Lee (1998), Jung and Klein (2001, 2005) and Esmaeili (2009) applied GP in order to analyze their problem. For extensive discussion of GP, refer to Duffin et al. (1967), Beightler (1976), Dembo (1982) and Boyd et al. (2005). One of the important issues missing from the literature is the impact of reliability on pricing and marketing issues. In other words, most of the above articles assume that items are produced by a perfect reliability. However, this assumption does not hold for real cases in which some of the defective goods are discarded. Besides, earlier works assume that holding cost per product and the setup cost per production cycle are known in advance, whereas these production components should be determined in coordination with pricing and marketing strategies. Also, most of the other papers fail to take account of interest and depreciation costs which cannot be ignored easily in many companies.

In this study, a new method is being proposed which incorporates the reliability as part of an integrated model. The model simultaneously determines price of products, marketing expenditure, lot size, setup cost, inventory holding cost and reliability of the production process. The objective is to minimize total costs including marketing, production, setup, holding, and interest and depreciation costs. This model is formulated in GP form and the optimal solution in closed form is determined using the art of GP technique. The study analyzes the behaviour of the model using some realistic example when different parameters are changed. This paper is organized as follows. The study first presents the problem formulation and necessary notations. The optimal solution is presented in the following section. Finally, concluding remarks are presented at the end to summarize the contribution of the work.

## PROBLEM STATEMENT

Consider a single product where demand is a function of selling price and marketing expenditure. Let  $P$ ,  $a$ ,  $M$  and

$b$  be the selling price per unit, price elasticity of demand, marketing expenditure per unit and marketing expenditure elasticity of demand, respectively. The study assumes

$$D = kP^{-a} M^b \quad (1)$$

Where, demand ( $D$ ) is defined as a function of price per unit ( $P$ ) with  $a > 1$  and  $0 < b < 1$ . The scaling constant  $k$  represents other related factors and the assumption  $a > 1$  implies that  $D$  increases at a diminishing rate as  $P$  decreases. This type of relationship is commonly used by many people in the literature (Fathian et al., 2009). In addition, (1) can be easily estimated by applying linear regression to the logarithm of the function. Let  $C$  be the production cost per unit. The study assumes that unit production cost ( $C$ ) can be discounted with  $c$ . Therefore,

$$C = rQ^{-c} \quad (2)$$

Where,  $Q$  is production lot size (units),  $r$  is the scaling constant for unit production cost. The exponent  $c$  represents lot size elasticity of production unit cost with  $0 < c < 1$  which is almost the same as price elasticity  $a$  and the study considers a small value for it, say  $c = 0.02$ .

Let  $T(R,S,H)$  denotes the total cost of interest and depreciation. Also,  $R$ ,  $S$  and  $H$  indicate reliability of production, setup cost per production cycle and inventory holding cost per product per unit time, respectively. Then, the total cost of interest and depreciation per production cycle is assumed as a function of reliability, setup and holding costs according to:

$$T(R,S,H) = IR^d S^{-e} H^{-f} \quad (3)$$

where  $d$ ,  $e$  and  $f$  are positive parameters ( $d, e, f \geq 0$ ) and represent reliability elasticity, setup cost elasticity and inventory holding cost elasticity of  $T(R,S,H)$ , respectively. This function is similar to the functions considered by Van Beek and Putten (1987) and Leung (2007).

Equation (3) demonstrates that an increase in the reliability of the production process leads to growth in total interest and depreciation cost. This relationship can be realized easily, regarding the fact that high reliability can only be achieved with additional cost in practice. In other words, usually significant investment is needed to improve the reliability of the production which can be expressed as expected fraction acceptable. Consequently,

interest and depreciation cost of the high reliability process is much more than low level one.

Equation (3) also implies that when the setup and holding costs are decreased the total costs of interest and depreciation also increase. This relationship is apparent and is based on the fact that manufacturers must invest heavily in order to reduce the setup cost per production and the inventory holding cost per product. For instance, it may cost us much more to decrease the unit setup cost, since the study needs to acquire expensive facilities. Similarly, it needs to spend more in order to decrease the deterioration and inventory holding cost per product.

### THE PROPOSED MODEL

In this section, the study proposes an integrated model which simultaneously determines price ( $P$ ), lot size ( $Q$ ), marketing expenditures ( $M$ ), production reliability ( $R$ ), setup cost ( $S$ ) and holding cost ( $H$ ). The objective function is to maximize profit as follows:

Profit = Sales revenue - Marketing cost

- Production cost - Setup cost
- Holding cost - Interest and Depreciation cost

Note that only  $R\%$  of total products are acceptable and  $(1 - R)\%$  of the produced items are defective and must be discarded. It means

that  $\frac{D}{R}$  products must be produced to satisfy the whole demand. In

addition, it implies that the length of a production cycle is  $\frac{QR}{D}$  and

the number of cycles per year is  $\frac{D}{QR}$ . Also, the average amount of

inventory held each year is  $\frac{QR}{2}$ . Therefore, the model can be

formulated as follows:

$$\text{Max } (P, Q, M, R, S, H) = PD - \frac{MD}{R} - \frac{CD}{RQR} - \frac{SD}{RQR} - \frac{HQR}{2} - T(S, R, H) \frac{D}{QR} \quad (4)$$

subject to :  $R \leq v$   
 $0 < P, Q, M, R, S, H$

where  $(P, Q, M, R, S, H)$  denotes the total profit and  $v$  is the upper bound for the reliability. The constraint of the model ( $R \leq v$ ) indicates that the reliability of the system is limited and cannot exceed  $v$ . It is obvious that this constraint can be written in the form of  $\frac{R}{v} \leq 1$ .

The proposed model (4) is a constrained signomial GP with zero degree of difficulty. As the global optimality is not guaranteed for a signomial problem (Duffin et al., 1967), the study modify (4) into the posynomial GP problem. It is assumed that there is a lower bound  $Z$

for the objective function of (4) such that maximizing  $Z$  (or minimizing  $Z^{-1}$ ) is equivalent to maximizing the objective value. By this approach the signomial problem (4) is transformed into the problem (5) with an extra constraint and decision variable  $Z$ :

Min  $Z^{-1}$  (or Max  $Z$ )  
 subject to :  
 $PD - \frac{MD}{R} - \frac{CD}{R} - \frac{SD}{QR} - \frac{HQR}{2} - T(S, R, H) \frac{D}{QR} \geq Z$  (5)  
 $\frac{R}{v} \leq 1$   
 $0 < P, Q, M, R, S, H, Z$

Since  $Z > 0$ ; the first constraint of (5) can be transformed into:

$$P^{-1}D^{-1}Z + P^{-1}R^{-1}M + CP^{-1}R^{-1} + SP^{-1}Q^{-1}R^{-1} + \frac{HQR}{2}P^{-1}D^{-1} + T(S, R, H)P^{-1}Q^{-1}R^{-1} \leq 1 \quad (6)$$

Note that (6) can be easily obtained by dividing the first constraint of (5) by total revenue,  $PD$ , and rearranging the terms.

Substituting (1) - (3) into (6), results in constraint (7) are as follows:

$$k^{-1}P^{a-1}M^{-b}Z + P^{-1}R^{-1}M + rQ^{-c}P^{-1}R^{-1} + SP^{-1}Q^{-1}R^{-1} + \frac{k^1HQR}{2}P^{a-1}M^{-b} + IP^{-1}Q^{-1}R^{d-1}S^{-e}H^{-f} \leq 1 \quad (7)$$

Considering (5) and (7), finally, signomial problem (4) is transformed into:

Min  $Z^{-1}$  (or Max  $Z$ )  
 subject to :  
 $k^{-1}P^{a-1}M^{-b}Z + P^{-1}R^{-1}M + rQ^{-c}P^{-1}R^{-1} + SP^{-1}Q^{-1}R^{-1} + \frac{k^1HQR}{2}P^{a-1}M^{-b} + IP^{-1}Q^{-1}R^{d-1}S^{-e}H^{-f} \leq 1$  (8)  
 $\frac{R}{v} \leq 1$   
 $0 < P, Q, M, R, S, H, U$

The model (8) is a posynomial GP problem with zero degree of difficulty. Therefore, this problem can be solved globally by its dual problem (see Duffin et al., 1967). The dual problem of (8) is formulated as follows:

$$\text{Max } F(W) = \frac{1}{w_0} \frac{k^{-1}}{w_1} \frac{1}{w_2} \frac{1}{w_3} \frac{1}{w_4} \frac{1}{w_5} \frac{1}{w_6} \frac{1}{w_7} \frac{1}{w_8} \frac{1}{w_9} \frac{1}{w_{10}} \frac{1}{w_{11}} \frac{1}{w_{12}} \frac{1}{w_{13}} \frac{1}{w_{14}} \frac{1}{w_{15}} \frac{1}{w_{16}} \frac{1}{w_{17}} \frac{1}{w_{18}} \frac{1}{w_{19}} \frac{1}{w_{20}} \frac{1}{w_{21}}$$

Subject to:

$$w_0 = 1$$

$$\begin{aligned}
(a-1)w_{11} - w_{12} - w_{13} - w_{14} + (a-1)w_{15} - w_{16} &= 0 \\
(-c)w_{13} - w_{14} + w_{15} - w_{16} &= 0 \\
(-b)w_{11} + w_{12} + (-b)w_{15} &= 0 \\
-w_{12} - w_{13} - w_{14} + w_{15} + (d-1)w_{16} + w_{21} &= 0 \\
w_{14} + (-e)w_{16} &= 0 \\
-w_0 + w_{11} &= 0 \\
w_{15} + (-f)w_{16} &= 0 \\
1 = w_{11} + w_{12} + w_{13} + w_{14} + w_{15} \\
+w_{16} &= w_{21} \\
w_{11}, w_{12}, w_{13}, w_{14}, w_{15}, w_{16}, w_{21} &\geq 0
\end{aligned}$$

Solving model (9) for the optimal solutions, leads to the following results (throughout this paper, \* denotes optimality):

$$\begin{aligned}
w_{15}^* &= \frac{cf(b+1-a)}{cf(-b-1+a) + (e-f+1) - c(f+1)} \\
w_{11}^* &= 1 \\
w_{12}^* &= b(1+w_{15}^*) \\
w_{13}^* &= \frac{-e+f-1}{fc} \\
w_{14}^* &= \frac{ew_{15}^*}{14f} \\
w_{16}^* &= \frac{w_{15}^*}{15f} \\
w_{21}^* &= \frac{(-e+f-1)(1-c) - c(d+fb)}{fc} + b \\
1 &= w_{11} + w_{12} + w_{13} + w_{14} + w_{15} + w_{16}
\end{aligned}$$

Note that in order to have a feasible dual solution, variables  $w_{11}, w_{12}, w_{13}, w_{14}, w_{15}, w_{16}$  and  $w_{21}$  must remain positive. As a result, we made the following assumptions:

$$\begin{aligned}
b-a+1 &< 0 \\
(e-f+1) - cf(b-a+1) - c(f+1) &< 0 \\
e-f+1 &< 0 \\
((e-f+1)(c-1) - cd)(b-a+1) + b(e-f+1) - bc &< 0
\end{aligned}$$

At this stage, for  $i = 1, 2$  and  $j = 1 \dots 6$  suppose there is,

$$w_{ij} = \frac{w_{ij}}{i} \quad (12)$$

Where,  $w_{ij}$  are the weights of the terms in the constraints of model

(8). In fact,  $w_{11}$  to  $w_{16}$  indicates the proportion of profit ( $w_{11}$ ), marketing cost ( $w_{12}$ ), production cost ( $w_{13}$ ), setup cost ( $w_{14}$ ), inventory cost ( $w_{15}$ ), and interest and depreciation cost ( $w_{16}$ ) to the total sales revenue, respectively. The following relations must hold:

$$\begin{aligned}
w_{11} &= k^{-1}P^{a-1}M^{-b}Z \\
w_{12} &= P^{-1}R^{-1}M \\
w_{13} &= rQ^{-c}P^{-1}R^{-1} \\
w_{14} &= SP^{-1}Q^{-1}R^{-1} \\
w_{15} &= \frac{k^{-1}HQRP^{a-1}M^{-b}}{2} \\
w_{16} &= IP^{-1}Q^{-1}R^{d-1}S^{-e}H^{-f} \\
w_{21} &= \frac{R}{V}
\end{aligned}$$

Using (13), the optimal solutions of the problem are summarized as follows,

$$\begin{aligned}
Q^* &= \frac{1}{-ce+e+1-c+caf-cf-bcf-f} \\
&\times \frac{-fb}{12} \frac{e-af+bf+f+1-e}{13} \frac{14}{15} \frac{d-af+2f}{16} \left( \frac{21}{21} \right) \\
P^* &= \frac{rV^{-1}Q^{-c}}{13} \\
M^* &= \frac{rQ^{-c}}{12} \\
R^* &= V \frac{21}{13} \\
S^* &= \frac{rQ^{-c+1}}{14} \\
H^* &= 2kr^{-a+b+1} \frac{b}{12} \frac{a-b-1}{13} \frac{V}{15} \frac{a-2}{21} Q^{ac-c-bc-1} \\
Z^* &= kr^{b-a+1} \frac{a-1}{11} \frac{b}{12} \frac{a-b-1}{13} \frac{a-1}{21} Q^{ac-bc-c}
\end{aligned}$$

## RESULTS AND DISCUSSION

In order to have a better understanding of the behavior of the algorithm used for the model presented in this paper, there is need to be carefully analyze some of the main parameters of the model. The study first explained the implementation of the method by presenting an example. Then analyzed the impacts of the changes in parameters on the optimal results is analyzed. Consider

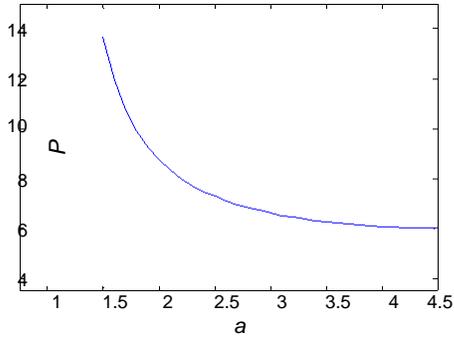


Figure 1. Changes in  $P$  with respect to change in  $a$ .

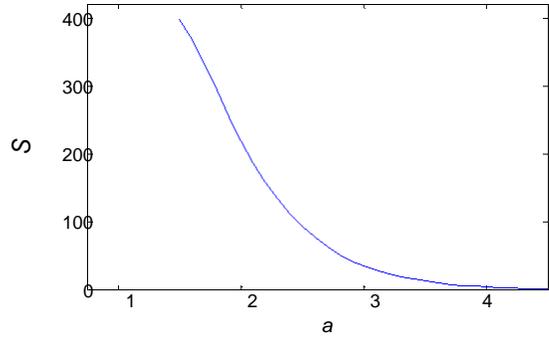


Figure 4. Changes in  $S$  with respect to change in  $a$ .

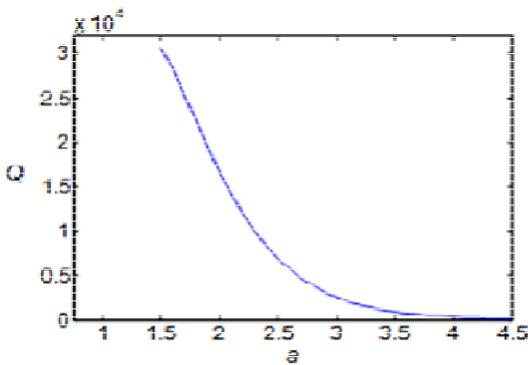


Figure 2. Changes in  $Q$  with respect to change in  $a$ .

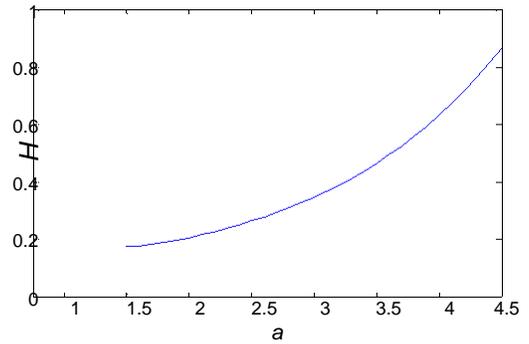


Figure 5. Changes in  $H$  with respect to change in  $a$ .

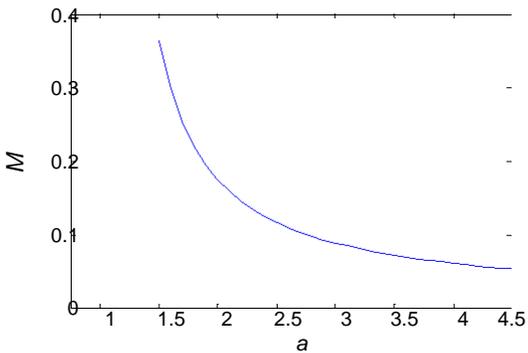


Figure 3. Changes in  $M$  with respect to change in  $a$ .

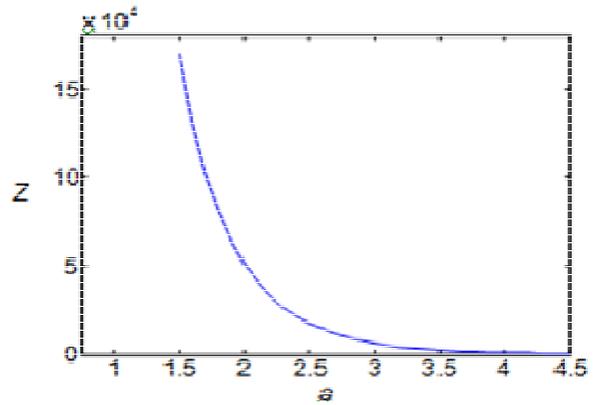


Figure 6. Changes in  $Z$  with respect to change in  $a$ .

the following data:

$$a = 3 \quad b = 0.05 \quad c = 0.02, \quad d = 1 \quad e = 1$$

$$f = 7 \quad k = 10^6 \quad r = 4 \quad v = 0.8 \quad l = 1$$

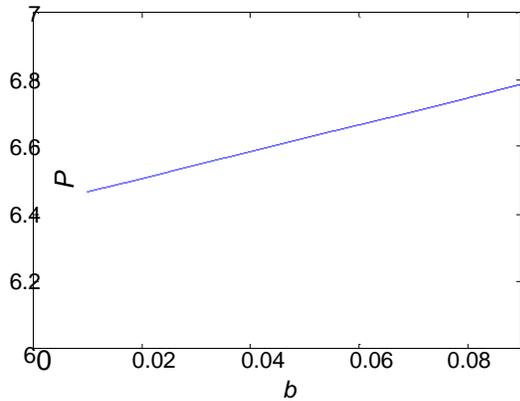
The optimal solution using the procedure explained in the previous section can be obtained easily as follows,

$$P^* = 6.6249 \quad Q^* = 2612.1 \quad M^* = 0.0883 \quad R^* = 0.8 \quad S^* = 35.7090$$

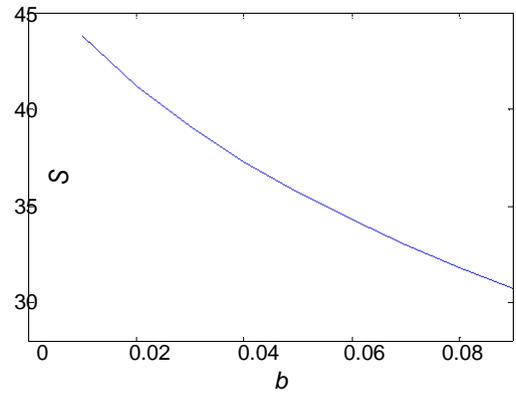
$$H^* = 0.3487 \quad Z^* = 6362.7$$

Now the effects of changes in parameters to the variations in the optimal solution are evaluated. In other words, the study develops some managerial insights by studying how the optimal solution would vary as the inputs values change. More specifically, the study examines the changes in  $P^*$ ,  $Q^*$ ,  $M^*$ ,  $S^*$ ,  $H^*$  and  $Z^*$  according to changes in the inputs  $a, b, c$  and  $v$ .

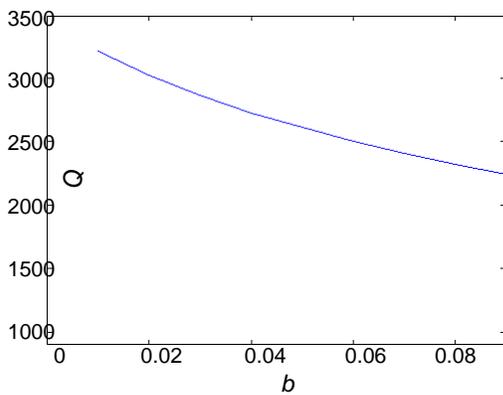
Figures 1 - 6 respectively reveal that any increase in price elasticity of demand ( $a$ ) leads to a lower optimal price ( $P$ ), smaller optimal production lot size ( $Q$ ), lower



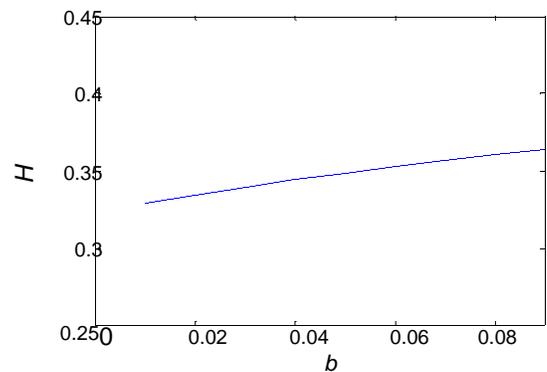
**Figure 7.** Changes in  $P$  with respect to change in  $b$ .



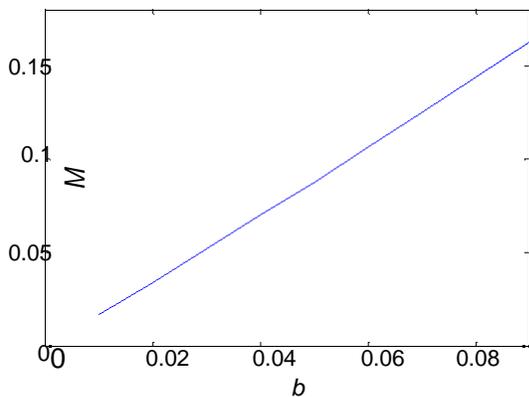
**Figure 10.** Changes in  $S$  with respect to change in  $b$ .



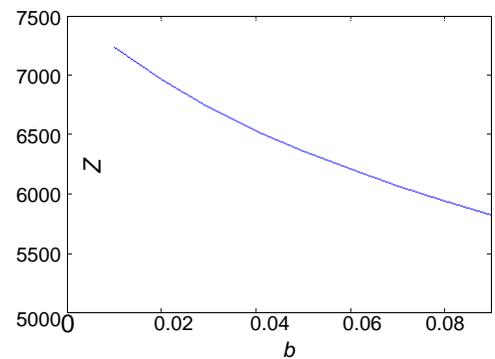
**Figure 8.** Changes in  $Q$  with respect to change in  $b$ .



**Figure 11.** Changes in  $H$  with respect to change in  $b$ .



**Figure 9.** Changes in  $M$  with respect to change in  $b$ .



**Figure 12.** Changes in  $Z$  with respect to change in  $b$ .

optimal marketing expenditure per unit ( $M$ ), lower optimal setup cost per production cycle ( $S$ ), higher inventory holding cost per product ( $H$ ) and lower profit ( $Z$ ).

Moreover, it follows from Figures 7 - 12 that any increase in marketing expenditure elasticity of demand ( $b$ ) results in a higher optimal price ( $P$ ), smaller optimal

production lot size ( $Q$ ), higher optimal marketing expenditure per unit ( $M$ ), lower optimal setup cost per production cycle ( $S$ ), higher inventory holding cost per product ( $H$ ) and lower profit ( $Z$ ), respectively. Figures 13 - 18, however, indicate that any increase in lot size elasticity of production unit cost ( $c$ ) decreases optimal price ( $P$ ), increases optimal production lot size ( $Q$ ), reduces optimal marketing expenditure per unit ( $M$ ),

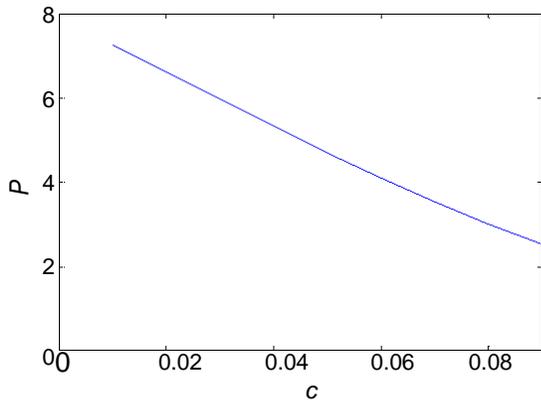


Figure 13. Changes in  $P$  with respect to change in  $c$ .

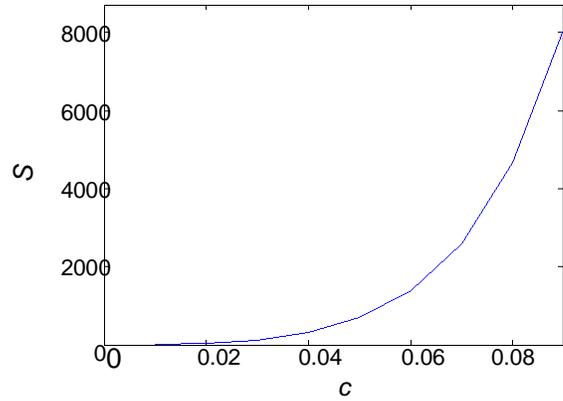


Figure 16. Changes in  $S$  with respect to change in  $c$ .

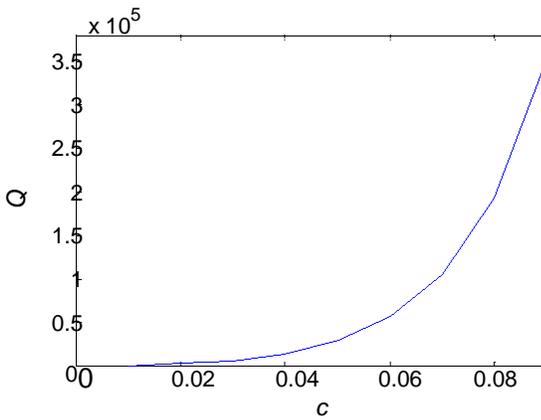


Figure 14. Changes in  $Q$  with respect to change in  $c$ .

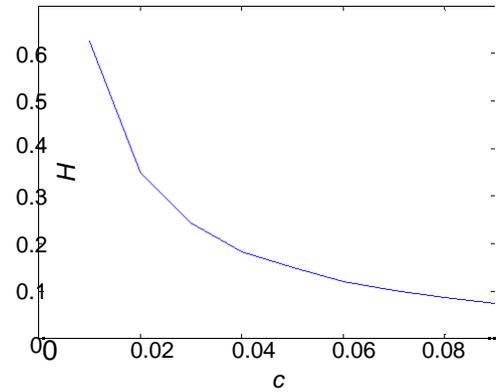


Figure 17. Changes in  $H$  with respect to change in  $c$ .

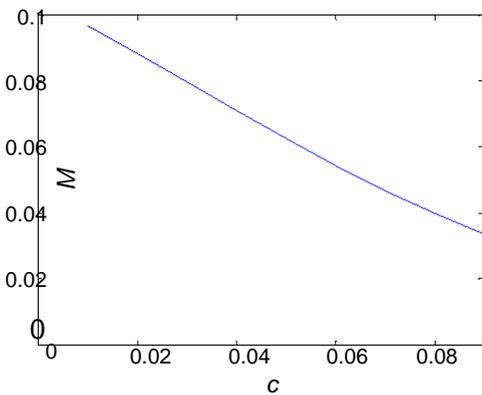


Figure 15. Changes in  $M$  with respect to change in  $c$ .

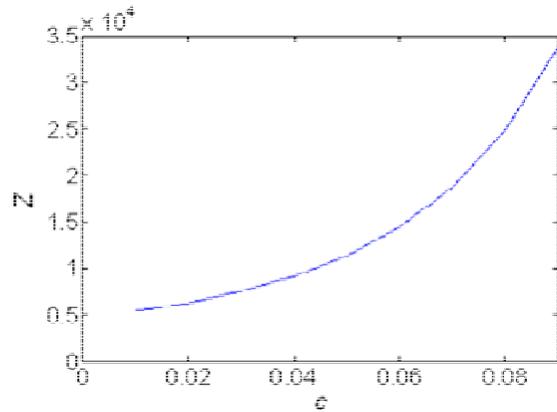


Figure 18. Changes in  $Z$  with respect to change in  $c$ .

raises optimal setup cost per production cycle ( $S$ ), reduces inventory holding cost per product ( $H$ ) and increases profit ( $Z$ ), respectively. Also, it can be seen from Figures 19 - 24 that an increase in maximum reliability of the reliability ( $v$ ) can lead to a lower optimal

optimal price ( $P$ ), larger optimal production lot size ( $Q$ ), lower optimal marketing expenditure per unit ( $M$ ), higher optimal setup cost per production cycle ( $S$ ), lower inventory holding cost per product ( $H$ ) and higher profit ( $Z$ ), respectively.

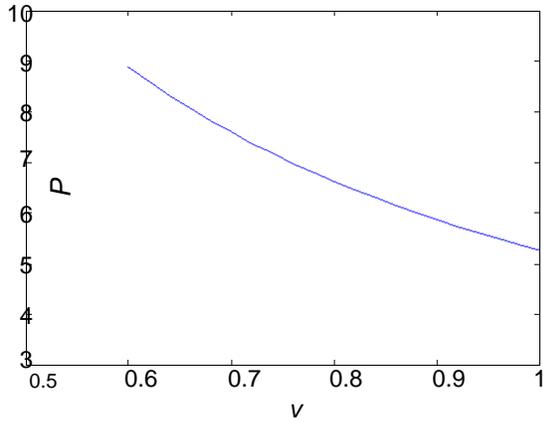


Figure 19. Changes in  $P$  with respect to change in  $v$ .

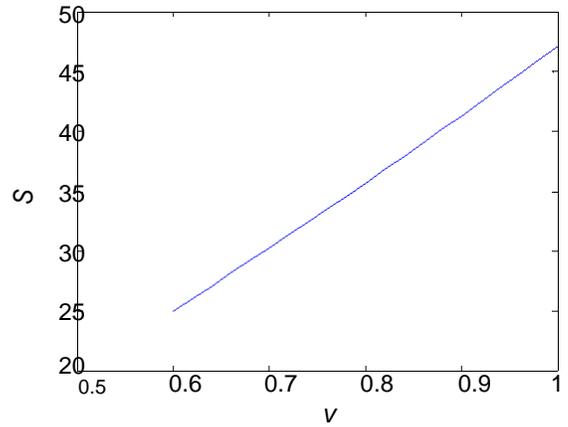


Figure 22. Changes in  $S$  with respect to change in  $v$ .

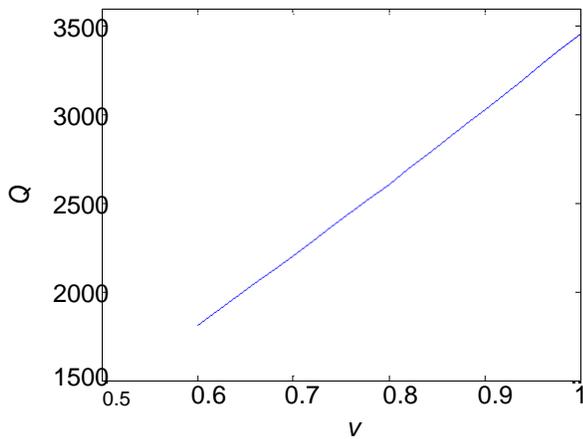


Figure 20. Changes in  $Q$  with respect to change in  $v$ .

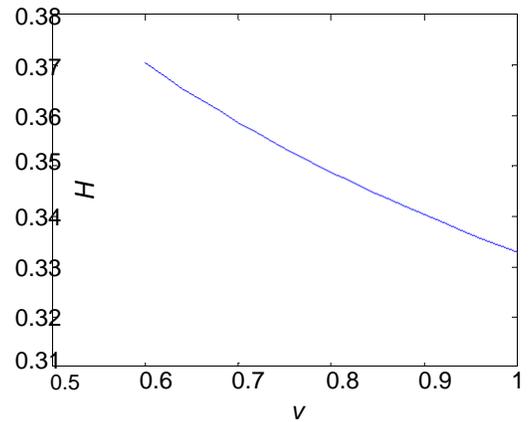


Figure 23. Changes in  $H$  with respect to change in  $v$ .

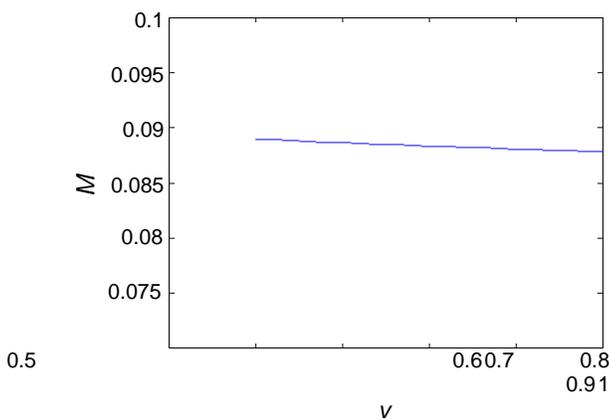


Figure 21. Changes in  $M$  with respect to change in  $v$ .

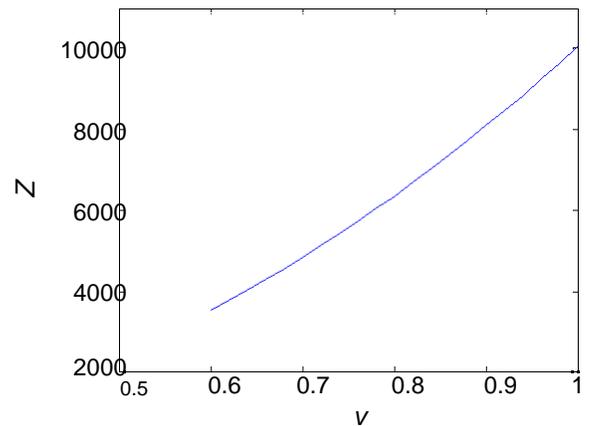


Figure 24. Changes in  $Z$  with respect to change in  $v$ .

## CONCLUSION AND FUTURE RESEARCH

The study has proposed a method to determine the

optimal marketing and production strategies for a product with different characteristics such as price and reliability. The proposed model of this paper is capable of maximizing

the total profit by changing the price, marketing expenditure and production decisions when the demand is affected by the price and the marketing expenditure. The model has been formulated as a nonlinear programming problem and geometric programming has been used to find the optimal solution in closed form. The implementation of the presented approach has been illustrated using a fairly simple and practical example. The paper have also studied the behaviour of the optimal price, the marketing expenditure, the lot size, the set up cost, the holding cost and the total profit when the market and the production conditions such as the marketing elasticity of demand and the lot size elasticity of production unit cost are changed. This research can be extended in some directions. First, it would be interesting to model the problem when various products are being sold in several markets. Besides, the model can be expanded to include more parameters like lost sales cost and production rate. Another extension for this study is the consideration of the problem in uncertain environment; for instance, when the parameters are random or fuzzy.

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