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Real estate investment in seismically active regions: Feasibility assessment and decision making

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Lots of major cities worldwide, such as Tokyo, Los Angeles, Christchurch, Taipei etc., are located in the seismically active regions. These cities have suffered damages from a number of earthquakes throughout their histories. Due to potential earthquake hazards, the security of real estate investment in these cities is apparently different from those in other regions, and the investment profits may be corroded due to earthquake disasters. The 1999 Chi-Chi earthquake which occurred in Taiwan is an actual example (MCEER, 2000). This earthquake caused varying degrees of damage to more than 10,000 buildings and loss to a great number of investors of real estate and house owners in the disaster area. From the viewpoint of financial analysis, a complete model should give full consideration to the possible future risky cash flows. Returns of real estate investments in seismically active regions in the long run may be more relevantly evaluated, provided that the uncertainties of both economic fluctuation and potential cost to repair earthquake damages are properly taken into account in analysis model. However, how to effectively incorporate the cost to repair earthquake damages into the analysis model exits a knowledge gap.

In the current real estate market, net present value (NPV) of the discounted cash flow (DCF) method may be one of the most commonly used indicators in cost-benefit analysis of real estate investment. The cash flow used by the traditional DCF method is constant, and the total present discounted value of cash flow during the period of investment will be correspondingly a constant value. However, cash flow in reality often shows stochastic volatility due to the impact of external factors. The argument for using such a method is that a single value may not provide the investors with sufficient information to make an informed decision (Brown, 1991; Dixit and Pindyck, 1994; Byrne and Cadman, 1996). Therefore, the traditional DCF method often cannot be directly used before they are amended or adjusted to meet actual demand. Furthermore, the general purpose of real estate investments is to gain profits from the subject matters of investment under acceptable levels of risk or reliability. Prior to investment decision making, prudent investors must address the questions: “How many profits they will gain from their investments in the future?” and “What is the probability (reliability) that they will gain the profits?”, and utilize modern analysis tools to assess feasibility for all candidate projects so as to select the relatively optimum one from feasible projects for investment.

Using the Monte Carlo simulation technique with NPV as the indicator, this study proposes a probabilistic
framework for constructing the relation curve between annual rate of return and reliability for the purpose of feasibility assessment and decision-making. In addition to fluctuation of rent market of real estate, the uncertainty induced by earthquake risk is also considered in this study. To clearly explain how to use the proposed framework and process to construct the relation curve of annual rate of return versus reliability and how to utilize the curve to assess feasibility of real estate investment and make decisions, this study takes three possible investment objects of real estate located at three major cities in Taiwan as an example.

**METHODOLOGY**

Using the probability-based DCF method (Nygard, 1999) with the Monte Carlo sampling technique (Rubinstein and Kroese, 2007), a framework for constructing the relation curve of annual rate of return and corresponding reliability of return on investment is proposed. Two random factors including fluctuation of rental income of real estate and random repair cost due to earthquake are introduced into the calculation of NPV. The framework is repeatedly performed for all candidate real estate objects. Then, the relation curves of annual rate of return and corresponding reliability for these objects are constructed. Figure 1 is a flowchart showing the procedure and the steps for constructing the curves. From the viewpoint of cost-benefit, the curves not only can be used to assess feasibility for all candidate real estate objects, but can be used as a tool to select the relatively best one from the feasible objects. The decision-making of investment is based on two criteria including required rate of return and reliability of return on investment, set by the investor. The reliability of return on investment represents the probability that the investor will gain at least the return on investment. As a real estate object satisfies these two criteria set by the investor at the same time, the object is acceptable and feasible. Then, the investor can make decision and choice the highest reliability object from the feasible candidates.

**Figure 1.** Flowchart showing the steps for constructing the relation curves of annual rate of return versus corresponding reliability for candidate real estate.

1. Construct fragility curves and introduce the hazard data of the building site
2. Calculate probability of annual repair cost for various damage states
3. Generate random values for PVRC
4. Generate random values for PVAT
5. Calculate NPV for N observations
6. Calculate probability distribution of total present value of the repair cost
7. Estimate probability distribution of total present value of the after-tax cash inflows
8. Plot the CDF graph of NPV
9. Estimate probability distribution of total present value of the after-tax cash inflows
10. Construct fragility curves and introduce the hazard data of the building site
11. Flowchart showing the steps for constructing the relation curves of annual rate of return versus reliability for candidate real estate objects

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Using the probability-based DCF method (Nygard, 1999) with the Monte Carlo sampling technique (Rubinstein and Kroese, 2007), a framework for constructing the relation curve of annual rate of return and corresponding reliability of return on investment is proposed. Two random factors including fluctuation of rental income of real estate and random repair cost due to earthquake are introduced into the calculation of NPV. The framework is repeatedly performed for all candidate real estate objects. Then, the relation curves of annual rate of return and corresponding reliability for these objects are constructed. Figure 1 is a flowchart showing the procedure and the steps for constructing the curves. From the viewpoint of cost-benefit, the curves not only can be used to assess feasibility for all candidate real estate objects, but can be used as a tool to select the relatively best one from the feasible objects. The decision-making of investment is based on two criteria including required rate of return and reliability of return on investment, set by the investor. The reliability of return on investment represents the probability that the investor will gain at least the return on investment. As a real estate object satisfies these two criteria set by the investor at the same time, the object is acceptable and feasible. Then, the investor can make decision and choice the highest reliability object from the feasible candidates.
Net present value

Various cash flows in transaction and holding period are considered in analysis. The cash inflows include mortgage loan at initial stage, annual rental incomes and quick liquidation value of real estate at the end of investment period; the cash outflows include initial purchase price of real estate, transaction cost at initial stage (breakage fee and deed tax), annual payment for principal and interest, annual income tax, annual land tax and building tax, annual operating expense, annual cost to repair earthquake damages and transaction cost at the end (breakage fee and increment tax on land value). The quick liquidation value of real estate at the end of investment period allows for depreciation of buildings and appreciation of land value. The formula of NPV can be given as:

\[
NPV(q) = LA - IIC + \sum_{t=1}^{T} \left( \frac{AT_t}{1+q} \right) - \sum_{t=1}^{T} \left( \frac{RC_t}{1+q} \right),
\]

where, \( LA \) refers to loan amount, \( IIC \) initial investment cost, \( AT_t \) cash flow after taxes at the \( t \) year, \( q \) annual discount rate, \( T \) assets holding period, \( RC_t \) earthquake repair cost at the \( t \) year, \( LVT_t \) quick liquidation value at the \( T \) year, \( LVIT_T \) increment tax on land value, \( PP_t \) loan principal repayment at the \( t \) year and \( BF_T \) breakage fee paid for selling real estate held for \( T \) years. In Equation 1, initial investment cost \( IIC \) equals the sum of the initial purchase price of real estate \( IPP \), deed tax \( DT \) and brokerage fee paid for purchase of real estate \( BF_0 \); the cash flow after taxes at the \( t \) year \( AT_t \) can be given as:

\[
AT_t = BT_t - LVT_t - HT_t - ICT_t
\]

where \( BT_t \) is cash flow before taxes at the \( t \) year, \( LVT_t \) is land value tax, and \( HT_t \) is house tax. The income tax at the \( t \) year \( ICT_t \) equals cash flow before taxes \( BT_t \) minus deducted tax exemptions \( DE_t \) and then multiplied by income tax rate \( \chi \). In this study, deducted tax exemptions \( DE_t \) is taken as \( 0.43BT_t \), including operation costs for rent, cost for assets maintenance and improvement and so on. The cash flow before taxes \( BT_t \) can be given as:

\[
BT_t = NI_t - PP_t - IP_t
\]

where \( PP_t \) is loan principal payment and \( IP_t \) is payment for annual loan interest. The net rental income \( NI_t \) at the \( t \) year can be given as:

\[
NI_t = (1 - \nu_t) PGI_t - OE_t = (1 - k)(1 - \nu_t) PGI_t
\]

where, \( PGI_t \) refers to potential gross rental income at \( t \) year, \( \nu_t \) vacancy rate, \( OE_t \) operation cost at \( t \) year, \( k \) operating expenditure ratio.

Reliability of return on investment

If the total present value of all cash inflows in Equation 1 is designated as random variable \( NPVI \); and the total present value of all cash outflows is designated as random variable \( NPVO \), then Equation 1 can be rewritten as:

\[
NPV(q) = NPVI(q) - NPVO(q)
\]

It is noted that \( NPV(q) \) is generally a key output variable which is used to summarize the net returns for a multi-year investment into a single variable under the preset discount rate \( q \). When \( NPV(q) \) is larger than zero, the rate of return of the investment will exceed the preset discount rate \( q \); on the contrary, when \( NPV(q) \) is less than zero, the rate of return of the investment will be less than the preset discount rate \( q \). To conduct reliability analysis, \( NPV(q) \) may be referred to as performance function of real estate. If the rate of return on investment is specified as \( q \), the reliability \( R(q) \) can be defined as the probability that \( NPV(q) > 0 \) or \( NPVI(q) > NPVO(q) \), that is,

\[
R(q) = P(NPV(q) > 0) = P(NPVI(q) > NPVO(q))
\]

If the joint probability density function (JPDF) of \( NPVI \) and \( NPVO \) is known, it is possible to calculate the
Potential gross rental income

In this study, it is assumed that the potential gross rental income is a random walk process, and the potential gross rental income at the t year (PGI_t) is related to the potential gross rental income at the t-1 year (PGI_{t-1}). The relation between PGI_t and PGI_{t-1} can be given by:

$$PGI_t = PGI_{t-1} + Z_t$$  \hspace{1cm} (7)$$

where Z_t is the annual increment of potential gross rental income at the t year. The annual increments Z_t (t = 1,2,...,T) are mutually independent, and follow a normal distribution with mean value \( \mu \) and variance \( \sigma^2 \). Both \( \mu \) and \( \sigma^2 \) are constant. It is noted that annual increment \( Z_t \) generally depends on macroeconomic factors (for example, GDP increment and inflation rate) and characteristic conditions of real estate (for example, location, building material and construction years).

Equation 7 can be further rewritten as:

$$PGI_t = PGI_0 + \sum_{i=1}^{t} Z_i$$  \hspace{1cm} (8)$$

where, \( PGI_0 \) is the initial annual potential gross rental income.

Total present value of net rental income after taxes

In this study, the total present value of net rental income after taxes in Equation 1 is expressed as random variable PVAT. Thus:

$$PVAT = \alpha PGI_0 \sum_{t=1}^{T} \left( \frac{1}{1 + q} \right)^t + \alpha \sum_{t=1}^{T} \left( \frac{1}{1 + q} \right)^{t-1} \sum_{i=1}^{t} \beta (PP_t + IP_t) + LVT_t + HT_t \right.$$  \hspace{1cm} (10)$$

where \( \alpha = (1-k)(1-\nu) \beta ; \beta = 1 - \chi(1-0.43) \). From Equation 10, PVAT is a linear combination of \( Z_t \), \( t = 1,2,...,T \). It is easy to show that PVAT follows a normal distribution, and the mean value and variance can be given as:
The total present value of repair cost for earthquake damage in Equation 1 is expressed by random variable \( PVRC \): \[
E[PVAT] = \sum_{t=1}^{T} \frac{E[AT_t]}{1+q} = \alpha PGI \sum_{t=1}^{T} \frac{1}{1+q} + \beta \sum_{t=1}^{T} \frac{1}{(1+q)^t} - \sum_{t=1}^{T} \frac{\beta PP + \beta LP + LVT + HT}{(1+q)^t}.
\] (11)

In this study, the annual costs to repair earthquake damages \( RC_t \) (\( t = 1, 2, \ldots, T \)) are assumed to be independent and identically distributed (i.i.d.). Table 1 shows the probability distribution and all possible outcomes of \( PVRC \) for the case of 3-year holding period. For this case, the number of outcomes of \( PVRC \) is \( 5^3 \) or 125. As suggested by FEMA (1999), four limit states (\( LS \)) including limit states of slight, moderate, extensive and complete (that is, \( LS = ls, lm, le \) and \( lc \)) and five damage states (\( DS \)) including damage states of none, slight, moderate, extensive and complete (that is, \( DS = dn, ds, dm, de \) or \( dc \)) are considered in this study. Corresponding annual repair cost (\( RC \)) are \( cn \) for none damage, \( cs \) for slight damage, \( cm \) for moderate damage, \( ce \) for extensive damage and \( cc \) for complete damage (that is, \( RC = cn, cs, cm, ce \) or \( cc \)). It is widely accepted that the mathematical probability theory is a rational and natural basis for the modeling of structural

<table>
<thead>
<tr>
<th>No.</th>
<th>Sample point</th>
<th>( PVRC )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>none, none, none</td>
<td>0</td>
<td>( P (RC = cn)^3 )</td>
</tr>
<tr>
<td>2</td>
<td>{none, none, slight}</td>
<td>( \frac{cs}{1+q} )</td>
<td>( P (RC = cn)^2 P (RC = cs) )</td>
</tr>
<tr>
<td>3</td>
<td>{none, none, moderate}</td>
<td>( \frac{cm}{1+q} )</td>
<td>( P (RC = cn)^2 P (RC = cm) )</td>
</tr>
<tr>
<td>4</td>
<td>{none, none, extensive}</td>
<td>( \frac{ce}{1+q} )</td>
<td>( P (RC = cn)^2 P (RC = ce) )</td>
</tr>
<tr>
<td>5</td>
<td>{none, none, complete}</td>
<td>( \frac{cc}{1+q} )</td>
<td>( P (RC = cn)^2 P (RC = cc) )</td>
</tr>
<tr>
<td>6</td>
<td>{none, slight, none}</td>
<td>( \frac{cs}{1+q} )</td>
<td>( P (RC = cn)^2 P (RC = cs) )</td>
</tr>
<tr>
<td>124</td>
<td>complete, complete, extensive</td>
<td>( \frac{cc}{1+q} + \frac{cc}{1+q} + \frac{ce}{1+q} )</td>
<td>( P (RC = cc)^2 P (RC = ce) )</td>
</tr>
<tr>
<td>125</td>
<td>complete, complete, complete</td>
<td>( \frac{cc}{1+q} + \frac{cc}{1+q} + \frac{cc}{1+q} )</td>
<td>( P (RC = cc)^3 )</td>
</tr>
</tbody>
</table>

**Total present value of repair cost**

The total present value of repair cost for earthquake damage in Equation 1 is expressed by random variable \( PVRC \):
failure problems (Der Kiureghian, 1981; Shinozuka, 1983; Ellingwood, 2001). Then, the probability of annual repair cost for the damage states can be evaluated by total probability theory and given by:

\[
P(\text{RC} = c) \equiv P(DS = d) \equiv \sum_i P\{DS = d | LS = l_i\} \sum_i P\{LS = l | SI = a_i\} \Delta G(a_i)
\]

where the seismic intensity \(SI\) is characterized by the peak ground acceleration (PGA) of earthquake, and for convenience of numerical calculation, the values \(a_1\), \(a_2\), \(a_3\),..., that provide a suitable discretization of the continuous intensity parameter are adopted; \(\Delta G(a_i)\) is the absolute value of the increment of the seismic hazard curve between \(a_i\) and \(a_i + \Delta a\), and expresses the annual occurrence probability of a ground motion with intensity between \(a_i\) and \(a_i + \Delta a\); the conditional probability, \(P\{DS = d | SI = a_i\}\), of a building being in damage state \(d\) given the PGA of \(a_i\) can then simply be obtained from the difference between the fragilities of the bordering limit states of damage state \(d\) (Figure 3).

It is noted that the fragility curve is modeled commonly by a lognormal cumulative distribution function, defined by the median value (50\(^{th}\) percentile fragility), \(M\), and a logarithmic standard deviation, \(\beta\), of the intensity of ground shaking, \(a\). The fragility curve is described by:

\[
F(a | M, \beta) = \Phi(\ln(a / M) / \beta)
\]

in which \(\Phi[.]\) = the standard normal probability integral; \(M\) and \(\beta\) are two parameters of the fragility curve. The parameter values of \(M\) and \(\beta\) can refer to the study of Liao et al. (2006).

**EXAMPLE**

An investor would like to make an investment into commercial real estate in Taiwan. There are three investment objects (real estates A, B and C) shown in Table 2 to be considered. Real estates A, B and C are located at the cities of Taipei, Taichung and Kaohsiung, respectively. The buildings of real estates A, B and C, called buildings A, B and C, are 4-storey, 3-storey and
Table 2. Parameter values of real estates A, B and C used in modelling.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Real estate A</th>
<th>Real estate B</th>
<th>Real estate C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Taipei city</td>
<td>Taichung city</td>
<td>Kaohsiung city</td>
</tr>
<tr>
<td>Building type</td>
<td>4-storey reinforced concrete building</td>
<td>3-storey reinforced concrete building</td>
<td>5-storey reinforced concrete building</td>
</tr>
<tr>
<td>Building price (Million NT dollars)</td>
<td>22.50</td>
<td>9.00</td>
<td>11.25</td>
</tr>
<tr>
<td>Land price (Million NT dollars)</td>
<td>27.50</td>
<td>11.00</td>
<td>13.75</td>
</tr>
<tr>
<td>Initial purchase price, ( IPP ) (Million NT dollars)</td>
<td>50.00</td>
<td>20.00</td>
<td>25.00</td>
</tr>
<tr>
<td>Replacement cost (Million NT dollars)</td>
<td>29.25</td>
<td>11.70</td>
<td>14.04</td>
</tr>
<tr>
<td>Vacancy rate, ( \nu ) (%)</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Land increment rate (%)</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Mean of annual increment of potential gross rental income ( \mu )</td>
<td>( 0.02P_{GI_0} )</td>
<td>( 0.02P_{GI_0} )</td>
<td>( 0.01P_{GI_0} )</td>
</tr>
<tr>
<td>Std. Dev. of annual increment of potential gross rental income ( \sigma )</td>
<td>( 0.1P_{GI_0} )</td>
<td>( 0.2P_{GI_0} )</td>
<td>( 0.15P_{GI_0} )</td>
</tr>
</tbody>
</table>

5-storey reinforced concrete buildings built with moderate-code, high-code and moderate-code seismic standards in 1998, 2007, and 2003, respectively. The required annual rate of return and the corresponding reliability of return on investment are, respectively, set to be 4% and 0.55 by the investor. Parameter values used in modelling are shown in Table 2. If the down payment is 10 million NT dollars, and allowing for annual rate of interest of 3% and 20-year home mortgage loan, the loan ratios of real estates A, B and C are 80, 50 and 60%, respectively. The holding period is specified to be 3 years. The brokerage fee equals 5% of initial purchase price of real estate (buyer 2%, seller 3%), the operating ratio \( k \) is 0.5%. In addition, according to the Taiwan’s tax laws, the deducted exemption \( DE_I \) is taken as, \( 0.43BT_I \) deed tax 6%, income tax 12%, house tax 3%, land tax 1%, increment tax on land value 20%, and annual depreciation rate of buildings 1%.

Based on the investigated results proposed by Liao et al. (2006), the values of parameters \( M \) and \( \beta \) of fragility curves for various limit states for buildings A, B and C are tabulated in Table 3. Using Equation 15 with the parameter values shown in Table 3, the fragility curves for buildings A, B and C can be obtained. For simplicity, only the fragility curves for building A are shown in Figure 3. Figure 4 shows the seismic hazard curves for the cities of Taipei, Taichung and Kaohsiung. As shown in Figure 4, for a specified PGA, the Taichung city possesses the highest seismic risk, the Taipei city ranking after and the Kaohsiung city the lowest. Substituting the data of the hazard curves in Figure 4 and the fragility curves of buildings A, B and C into Equation 14, the occurrence probabilities of annual repair costs with respect to various damage states for buildings A, B and C can be estimated. In this study, the repair costs were taken as 0, 2, 10, 50, and 100% of the replacement cost for damage states of none, slight, moderate, extensive and complete, respectively, as suggested by FEMA (1999). Figure 5 shows the occurrence probabilities of annual repair costs with respect to various damage states for buildings A, B and C. As expected, building B located at the highest seismic hazard area has the highest probability for various degrees of damage and building C has the lowest probability.

To establish the relation curve of annual rate of return versus reliability, this study considered 12 different annual rates of return (\( q = 0.01, 0.02, \ldots \)).
Table 3. Parameter values of fragility curves for various limit states for buildings A, B and C.

<table>
<thead>
<tr>
<th>Building</th>
<th>Slight</th>
<th>Moderate</th>
<th>Extensive</th>
<th>Collapse</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$\beta$</td>
<td>$M$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>A</td>
<td>0.28</td>
<td>0.5</td>
<td>0.6</td>
<td>0.68</td>
</tr>
<tr>
<td>B</td>
<td>0.33</td>
<td>0.5</td>
<td>0.53</td>
<td>0.45</td>
</tr>
<tr>
<td>C</td>
<td>0.28</td>
<td>0.5</td>
<td>0.6</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Figure 4. Seismic hazard curves for the cities of Taipei, Taichung and Kaohsiung, Taiwan.

and 0.12). For each real estate, the Monte Carlo sampling technique was repeatedly performed to generate 32,600 random samples of NPV for each annual rate of return, based on the probability distribution of $PVAT$ with the parameter values obtained from Equations 11 and 12, and the probability distribution of $PVRC$ obtained from Table 1 with the data of Figure 5. Then, the CDF graph of NPV for each real estate was plotted, and the reliability corresponding with the annual rate of return was estimated from the CDF graph. The process for all annual rates of return considered was repeated. Then, the relation curve of annual rate of return versus reliability for each real estate was constructed. Figure 6 shows the curves of annual rate of return versus reliability for real estates A, B and C. Two requirements (annual rate of return of 4% and reliability of 0.55) set by the investor were marked as point $P$ in Figure 6. Figure 6 shows that the curves of real estates A and B lie above point $P$ and the curve of real estate C lies below point $P$. As annual rate of return was fixed at 4%, the reliabilities of real estates A and B were greater than 0.55, and only the reliability of real estate C was lower than 0.55. This means that real estates A and B satisfied the requirements of the investor. The investment of real estates A or B is feasible. On the contrary, real estate C was unable to simultaneously satisfy the two requirements of the investor. The investment of real estate C is not feasible. Furthermore, as reliability was specified at 0.55, the rate of return of 7.7% for real estate A was significantly greater than the rate of return of 4.2% for real estate B. As a result, the investor can make decision to choose real estate A.

SUMMARY AND CONCLUSION

This study addressed the issue of uncertainty induced by earthquake risk in real estate investment, considered both fluctuation of rent market of real estate and random repair cost due to earthquake in analysis, and provided a decision making process of real estate investment in seismically active regions. An example of three mutually exclusive investment projects was presented to illustrate how to use the process to make feasibility assessment and select the
Figure 5. Comparisons of occurrence probabilities of annual repair costs with respect to various damage states for buildings A, B and C.

Figure 6. Relation curves of annual rate of return versus corresponding reliability for real estates A, B and C.

relatively optimum one to complete an investment decision, which simultaneously met investment conditions and investors' requirements. It is noted that the study presumed that the time series model of potential grossrental income followed a random walk process. This presumption may not conform to reality, but it is still possible to get the realizations from numerical simulation as long as the time series model of actual potential gross income is given. Therefore, the proposed process and the method of feasibility assessment and decision making can be still spread and applied.

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